

# Astrophysical and Cosmological Implications of Large Volume String Compactifications

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**ABSTRACT:** We study the spectrum, couplings and cosmological and astrophysical implications of the moduli fields for the class of Calabi-Yau IIB string compactifications for which moduli stabilisation leads to an exponentially large volume  $\mathcal{V} \sim 10^{15} l_s^6$  and an intermediate string scale  $m_s \sim 10^{11}$  GeV, with TeV-scale observable supersymmetry breaking. All Kähler moduli except for the overall volume are heavier than the susy breaking scale, with  $m \sim \ln(M_P/m_{3/2})m_{3/2} \sim (\ln(M_P/m_{3/2}))^2 m_{susy} \sim 500$  TeV and, contrary to standard expectations, have matter couplings suppressed only by the string scale rather than the Planck scale. These decay to matter early in the history of the universe, with a reheat temperature  $T \sim 10^7$  GeV, and are free from the cosmological moduli problem (CMP). The heavy moduli have a branching ratio to gravitino pairs of  $10^{-30}$  and do not suffer from the gravitino overproduction problem. The overall volume modulus is a distinctive feature of these models and is an  $M_{plank}$ -coupled scalar of mass  $m \sim 1$  MeV and subject to the CMP. A period of thermal inflation may help relax this problem. This field has a lifetime  $\tau \sim 10^{24}$  s and can contribute to dark matter. It may be detected through its decays to  $\gamma\gamma$  or  $e^+e^-$ . If accessible the  $e^+e^-$  decay mode dominates, with  $\text{Br}(\chi \rightarrow \gamma\gamma)$  suppressed by a factor  $(\ln(M_P/m_{3/2}))^2$ . We consider the potential for detection of this field through different astrophysical sources: the Milky Way halo, the diffuse cosmic background and nearby galaxy clusters and find that the observed gamma-ray background constrains  $\Omega_\chi \lesssim 10^{-4}$ . The decays of this field may generate the 511 keV emission line from the galactic centre observed by INTEGRAL/SPI.

**KEYWORDS:** Cosmological moduli problem. Flux compactifications. Cosmology.

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## 1. Introduction

It is an old and hard problem to connect string compactifications to observational physics. The principal difficulty is that the compactification energy scales are usually much larger than those directly accessible to experiment. However, we are helped by the fact that compactifications do have generic and model-independent features. One such feature is the presence of a moduli sector, consisting of many gravitationally coupled scalar fields. String moduli are naively massless particles and as such would give rise to unobserved fifth forces. It is therefore necessary that they receive a mass and the generation of flux-induced moduli potentials has been an active topic of research over the last few years (for review articles see [1, 2, 3, 4]).

As the moduli determine the vacuum structure, models with stabilised moduli are a prerequisite for doing string phenomenology. One direction of research, looking towards particle physics, has been to study the structure of supersymmetry-breaking terms that arises, as such terms can only be calculated once the vacuum has been identified. However, moduli can also play an important role in cosmology. Open and closed string moduli have recently been used to build inflation models within string theory. Moduli tend to be good candidates for inflatons, as they are flat prior to supersymmetry breaking and are ubiquitous in string models as scalar fields which interact gravitationally and are singlets under the standard model gauge group. If sufficiently long-lived, moduli could also contribute to dark matter. However, moduli also cause cosmological problems. Their relatively weak, gravitational-strength interactions imply that moduli are either stable or decay late in the history of universe, and in the presence of low-energy supersymmetry generic moduli either spoil nucleosynthesis or overclose the universe.

It is helpful to re-examine late-time (i.e. post-inflationary) modular cosmology in the context of the explicit models of moduli stabilisation that have been developed. Examples of work in this direction are [5, 6, 7, 8, 9]. In making contact with phenomenology one promising class of compactifications are the large-volume models developed in [10, 11]. These occur in flux compactifications of IIB string theory with D-branes and orientifold planes, with the consistent inclusion of both  $\alpha'$  and nonperturbative corrections. These models dynamically stabilise the volume at exponentially large values, allowing the generation of hierarchies. The gravitino and string scales are given by

$$m_{3/2} \sim \frac{M_P}{\mathcal{V}}, \quad m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}. \quad (1.1)$$

Here  $\mathcal{V}$  is the dimensionless volume - the physical volume is  $\mathcal{V}l_s^6 \equiv \mathcal{V}(2\pi\sqrt{\alpha'})^6$ . Thus a compactification volume of  $10^{15}l_s^6$ , corresponding to a string scale  $m_s \sim 10^{11}\text{GeV}$ , can generate the weak hierarchy through TeV-scale supersymmetry [12]. In these models other hierarchical scales also appear as different powers of the volume - for example the axionic scale appears as  $f_a \sim M_P/\sqrt{\mathcal{V}} \sim 10^{11}\text{GeV}$  [13] and the neutrino suppression scale as  $\Lambda \sim M_P/\mathcal{V}^{1/3} \sim 10^{14}\text{GeV}$  [14]. We will give a more detailed review of large-volume models in section 2.

The moduli for these models divide into two classes,  $\Phi$  and  $\chi$ , associated respectively with ‘small’ cycles and the overall volume. These have masses

$$m_\Phi \sim \ln(M_P/m_{3/2})m_{3/2}, \quad m_\chi \sim m_{3/2} \left( \frac{m_{3/2}}{M_P} \right)^{\frac{1}{2}}. \quad (1.2)$$

The requirement of TeV supersymmetry constrains the mass of the light modulus to be  $\sim 1\text{MeV}$ . The purpose of this paper is to perform a detailed study of the physics and couplings of these moduli, computing the decay modes and branching ratios. We will see that starting with a well-motivated stringy construction, with a moduli potential that naturally generates the weak hierarchy, gives results significant different from those obtained under assumptions of generic behaviour [6, 7, 15, 16, 17]. As a concrete example, the branching ratio  $\Phi \rightarrow \psi_{3/2}\psi_{3/2}$  is a factor  $10^{30}$  smaller than the  $\mathcal{O}(1)$  expectations of [6, 7].

The structure of this paper is as follows. In sections 2 and 3 we review the large-volume models and provide a precise computation of the masses and couplings of the moduli fields. These sections are more formal in nature and a reader more interested in the resulting phenomenology of the moduli can skip these sections and start at section 4, using the results of section 3 that are summarised in table 1. In section 4 we review the cosmological problems moduli can cause, while in section 5 we analyse the behaviour of the large-volume moduli in the early universe and how they affect reheating, the cosmological moduli problem and the gravitino overproduction problem. In section 6 we study the ability of the moduli to contribute to dark matter and examine the ability of the light modulus to contribute to the 511keV line.

This paper differs from most of the recent literature on moduli cosmology, which has concentrated on their potential role as inflatons. Here we will simply assume that inflation has occurred in the early universe and concentrate on the moduli cosmology in the post-inflationary era.

## 2. Large Volume Models

Large volume models originate in string theory, but here we view them simply as supergravity models. Their simplest avatar is that of compactifications on  $\mathbb{P}^4_{[1,1,1,6,9]}$ , which has two Kähler moduli, denoted by  $T_s = \tau_s + ib_s$  and  $T_b = \tau_b + ib_b$ . The ‘s’ and ‘b’ stand for ‘small’ and ‘big’. The Calabi-Yau volume is  $\mathcal{V} = \frac{1}{9\sqrt{2}} \left( \tau_b^{3/2} - \tau_s^{3/2} \right)$  [18]. The geometry should be thought of as analogous to a Swiss cheese - the small modulus controls the size of the hole and the big modulus the size of the cheese. In terms of these the Kähler potential and superpotential are<sup>1</sup>

$$\mathcal{K} = -2 \ln \left( \frac{1}{9\sqrt{2}} \left( \tau_b^{3/2} - \tau_s^{3/2} \right) + \frac{\xi}{2g_s^{3/2}} \right) \quad (2.1)$$

$$W = W_0 + A_s e^{-a_s T_s}. \quad (2.2)$$

Here  $\xi = \zeta(3)\chi(M)/(2\pi)^3$  is a constant entering the  $\alpha'$  correction (with  $\chi(M)$  the Euler number of the Calabi-Yau manifold) and  $g_s$  is the string coupling.  $W_0$  is  $\mathcal{O}(1)$  and is the tree-level flux superpotential that arises after stabilising the dilaton and complex structure moduli. For practical convenience in our computations, we will rewrite (2.1) and (2.2) as

$$\mathcal{K} = -2 \ln \left( \left( \tau_b^{3/2} - \tau_s^{3/2} \right) + \xi' \right) \quad (2.3)$$

$$W = W_0 + A_s e^{-a_s T_s}, \quad (2.4)$$

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<sup>1</sup>In these string models there are also complex structure moduli  $U$  and the dilaton  $S$ . Their scalar potential has been found to dominate at large volume unless they sit at their minimum [10]. This serves as a trapping mechanism for these fields. Even though they have masses of order the TeV scale and couple with gravitational strength [11], this trapping indicates that while the Kähler moduli roll through the scalar potential and could have coherent oscillations around their minima, the fields  $U$  and  $S$  energetically prefer to essentially sit at their minima and therefore do not cause a cosmological problem. In this note we will only study the cosmological implications of the Kähler moduli.

absorbing the overall factor of  $9\sqrt{2}$  into the value of  $W_0$  and  $A_s$  (so  $W_0 \rightarrow 9\sqrt{2}W_0$  and  $A_s \rightarrow 9\sqrt{2}A_s$ ). Clearly this does not alter the physics in any way. After extremising the axionic field, the supergravity scalar potential at large volumes is given by

$$V = \frac{8(a_s A_s)^2 \sqrt{\tau_s} e^{-2a_s \tau_s}}{3\tau_b^{3/2}} - \frac{4a_s A_s W_0 \tau_s e^{-a_s \tau_s}}{\tau_b^3} + \frac{\nu |W_0|^2}{\tau_b^{9/2}}, \quad (2.5)$$

where  $\nu = \frac{27\sqrt{2}\xi}{4g_s^{3/2}}$ .

This potential has been studied in detail in [10, 11, 19]. It has a non-supersymmetric AdS minimum at  $\mathcal{V} \sim e^{a_s \tau_s} \gg 1$  with  $\tau_s \sim \frac{\xi^{2/3}}{g_s}$ . This minimum has a negative cosmological constant of order  $\frac{1}{\mathcal{V}^3}$ . There exist various methods to introduce a positive energy to uplift this minimum to de Sitter [20, 21, 22], and the uplifted minimum is stable against tunnelling [23]. The physics presented in this paper is not significantly affected by the details of the uplift, and so we do not consider the uplift further.

The stabilised exponentially large volume can generate hierarchies. As the gravitino mass is given by

$$m_{3/2} = e^{\hat{K}/2} W = \frac{W_0}{\mathcal{V}}, \quad (2.6)$$

it follows that an exponentially large volume can lead to a gravitino mass exponentially lower than the Planck scale. This allows a natural solution of the hierarchy problem through TeV-scale supersymmetry breaking. It follows from (2.6) that a TeV-scale gravitino mass requires  $\mathcal{V} \sim 10^{15}$ . Through a detailed analysis of the moduli potential and the F-terms that are generated [24], it can in fact be shown that the scale of soft terms is lowered compared to the gravitino mass by a factor  $\ln(M_P/m_{3/2})$ , so

$$m_{soft} = \frac{m_{3/2}}{\ln(M_P/m_{3/2})}.$$

A sensible phenomenology therefore requires  $m_{3/2} \sim 20\text{TeV}$ .

The potential (2.5) generates masses for the moduli. Estimates of these masses can be computed using  $m_b^2 \sim \mathcal{K}_{bb}^{-1} \partial^2 V / \partial \tau_b^2$  and  $m_s^2 \sim \mathcal{K}_{ss}^{-1} \partial^2 V / \partial \tau_s^2$ , giving<sup>2</sup>

$$m_{\tau_b} \sim \frac{M_P}{\mathcal{V}^{3/2}}, \quad m_{\tau_s} \sim \frac{M_P \ln(M_P/m_{3/2})}{\mathcal{V}}.$$

The light field is associated with the modulus controlling the overall volume, whereas the heavy field is that associated with the small blow-up cycle. In section 3 we give a much more detailed analysis of the spectrum of moduli masses and couplings.

### 3. Moduli Properties and Couplings

In this section we describe how to canonically normalise the moduli and compute their masses and couplings to matter particles.

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<sup>2</sup>The axionic partners of  $\tau_b, \tau_s$  also receive masses after their stabilisation. The partner of  $\tau_s$  has a mass of the same order as  $\tau_s$  whereas the axionic partner of  $\tau_b$  is essentially massless. Being an axion, it does not couple directly to observable matter and therefore does not play a role in our cosmological discussion below.

### 3.1 Normalisation and Couplings to Photons

We assume the minimum of the moduli potential has been located. By writing  $\tau_i = \langle \tau_i \rangle + \delta\tau_i$ , we can always expand the Lagrangian about the minimum of the moduli potential. In the vicinity of the minimum, we can write

$$\mathcal{L} = \mathcal{K}_{i\bar{j}} \partial_\mu (\delta\tau_i) \partial^\mu (\delta\tau_j) - V_0 - (M^2)_{i\bar{j}} (\delta\tau_i) (\delta\tau_j) - \mathcal{O}(\delta\tau^3) - \kappa \frac{\tau_\alpha}{M_P} F_{\mu\nu} F^{\mu\nu}. \quad (3.1)$$

Here we take  $f_{U(1)} = \kappa \tau_\alpha$  where  $\kappa$  is a normalisation constant and  $\alpha$  labels one of the small four-cycles since we assume the standard model lives on a stack of D7 branes wrapping the small four-cycle.<sup>3</sup> To express the Lagrangian (3.1) in terms of canonically normalised fields, we require the eigenvalues and normalised eigenvectors of  $(\mathcal{K}^{-1})_{i\bar{j}} (M^2)_{\bar{j}k}$ . Anticipating our use of the  $\mathbb{P}_{[1,1,1,6,9]}^4$  model, we now specialise to a 2-modulus model, in which we denote  $\tau_1 \equiv \tau_b, \tau_2 \equiv \tau_s$ . This sets  $\alpha = 2 = s$  above. In this case we write the eigenvalues and eigenvectors of  $(\mathcal{K}^{-1})_{i\bar{j}} (M^2)_{\bar{j}k}$  as  $m_\Phi^2, m_\chi^2$ , and  $v_\Phi, v_\chi$  respectively, with  $m_\Phi > m_\chi$ . The eigenvectors are normalised as  $v_\alpha^T \cdot \mathcal{K} \cdot v_\beta = \delta_{\alpha\beta}$ .

We may rewrite the Lagrangian in terms of canonical fields  $\Phi$  and  $\chi$  defined by

$$\begin{pmatrix} \delta\tau_b \\ \delta\tau_s \end{pmatrix} = \begin{pmatrix} v_\Phi \end{pmatrix} \frac{\Phi}{\sqrt{2}} + \begin{pmatrix} v_\chi \end{pmatrix} \frac{\chi}{\sqrt{2}}. \quad (3.2)$$

Canonically normalising the  $U(1)$  kinetic term, the Lagrangian (3.1) can be written as

$$\mathcal{L} = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - V_0 - \frac{1}{2} m_\Phi^2 \Phi^2 - \frac{1}{2} m_\chi^2 \chi^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{(\Phi(v_\Phi)_s + \chi(v_\chi)_s)}{4\sqrt{2}\langle\tau_s\rangle M_P} F_{\mu\nu} F^{\mu\nu}.$$

The coupling of the two moduli  $\Phi$  and  $\chi$  to photons, which we denote by  $\lambda$ , is then given by

$$\begin{aligned} \lambda_{\Phi\gamma\gamma} &= \frac{(v_\Phi)_s}{\sqrt{2}\langle\tau_s\rangle}, \\ \lambda_{\chi\gamma\gamma} &= \frac{(v_\chi)_s}{\sqrt{2}\langle\tau_s\rangle}. \end{aligned} \quad (3.3)$$

Thus, given the moduli Lagrangian we can follow a well-defined procedure to compute the moduli couplings to photons.

The explicit forms of the matrices  $(\mathcal{K}^{-1})_{i\bar{j}}$  and  $(M^2)_{\bar{j}k}$  for the large volume models can be computed and are given in the appendix. Importantly, it follows from the expression for the moduli Kähler potential (2.1) that there is a small mixing between the moduli  $\tau_b$  and  $\tau_s$ , and the canonically normalised fields couple to matter living on both small and large cycles. The matrix  $\mathcal{K}^{-1} M^2$  takes the form:

$$\mathcal{K}^{-1} M^2 = \frac{2a_s \langle \tau_s \rangle |W_0|^2 \nu}{3\langle \tau_b \rangle^{9/2}} \begin{pmatrix} -9(1-7\epsilon) & 6a_s \langle \tau_b \rangle (1-5\epsilon+16\epsilon^2) \\ -\frac{6\langle \tau_b \rangle^{1/2}}{\langle \tau_s \rangle^{1/2}} (1-5\epsilon+4\epsilon^2) & \frac{4a_s \langle \tau_b \rangle^{3/2}}{\langle \tau_s \rangle^{1/2}} (1-3\epsilon+6\epsilon^2) \end{pmatrix}, \quad (3.4)$$

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<sup>3</sup>A D7 wrapping the large four-cycle would give rise to unrealistically small values of the gauge couplings ( $1/g^2 \sim \nu^{2/3} \sim 10^{10}$ ).

where  $\epsilon = (4a_s\langle\tau_s\rangle)^{-1}$  and the expressions are valid to  $\mathcal{O}(\epsilon^2)$  (there are also  $1/\mathcal{V}$  corrections, which are negligible). (3.4) has one large and one small eigenvalue, denoted by  $m_\Phi^2$  and  $m_\chi^2$ . Because  $m_\Phi^2 \gg m_\chi^2$ , we have at leading order in  $\epsilon$ :

$$m_\Phi^2 \simeq \text{Tr}(\mathcal{K}^{-1}M^2) \simeq \frac{8\nu|W_0|^2a_s^2\langle\tau_s\rangle^{1/2}}{3\langle\tau_b\rangle^3} = (2m_{3/2}\ln(M_P/m_{3/2}))^2 \sim \left(\frac{\ln\mathcal{V}}{\mathcal{V}}\right)^2 \quad (3.5)$$

$$m_\chi^2 \simeq \frac{\text{Det}(\mathcal{K}^{-1}M^2)}{\text{Tr}(\mathcal{K}^{-1}M^2)} \simeq \frac{27|W_0|^2\nu}{4a_s\langle\tau_s\rangle\langle\tau_b\rangle^{9/2}} \sim \mathcal{V}^{-3}/\ln\mathcal{V}. \quad (3.6)$$

We can see explicitly the large hierarchy of masses among the two observable particles, with  $\Phi$  heavier than the gravitino mass and  $\chi$  lighter by a factor of  $\sqrt{\mathcal{V}}$ . We have numerically confirmed the analytic mass formulae of (3.5) and (3.6).<sup>4</sup>

Finding the eigenvectors of  $\mathcal{K}^{-1}M^2$  and using (3.2) we can write the original fields  $\delta\tau_{b,s}$  in terms of  $\Phi$  and  $\chi$  (in Planck units) as:<sup>5</sup>

$$\delta\tau_b = \left(\sqrt{6}\langle\tau_b\rangle^{1/4}\langle\tau_s\rangle^{3/4}(1-2\epsilon)\right) \frac{\Phi}{\sqrt{2}} + \left(\sqrt{\frac{4}{3}}\langle\tau_b\rangle\right) \frac{\chi}{\sqrt{2}} \sim \mathcal{O}(\mathcal{V}^{1/6}) \Phi + \mathcal{O}(\mathcal{V}^{2/3}) \chi \quad (3.7)$$

$$\delta\tau_s = \left(\frac{2\sqrt{6}}{3}\langle\tau_b\rangle^{3/4}\langle\tau_s\rangle^{1/4}\right) \frac{\Phi}{\sqrt{2}} + \left(\frac{\sqrt{3}}{a_s}(1-2\epsilon)\right) \frac{\chi}{\sqrt{2}} \sim \mathcal{O}(\mathcal{V}^{1/2}) \Phi + \mathcal{O}(1) \chi$$

This shows, as expected, that  $\tau_b$  is mostly  $\chi$  and  $\tau_s$  is mostly  $\Phi$ . However there is an important mixing, which is subleading and has coefficients depending on different powers of the volume  $\mathcal{V}$ . This illustrates the fact that although the large modulus  $\tau_b$  has no couplings to photons, the light field  $\chi$ , although mostly aligned with  $\tau_b$ , does have a measurable coupling to photons due to its small component in the  $\tau_s$  direction. This  $\chi\gamma\gamma$  coupling is determined by the coefficient  $\frac{\sqrt{6}}{2a_s}$  in (3.7), which happens to be volume independent.

The  $\chi$  Lagrangian is therefore

$$\mathcal{L}_\chi = -\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\left(\frac{\sqrt{6}}{2a_s\langle\tau_s\rangle}\right)\frac{\chi}{M_P}F_{\mu\nu}F^{\mu\nu}. \quad (3.8)$$

The Planck mass dependence is here included for explicitness. Notice that the coupling of  $\chi$  to photons is not only suppressed by the Planck scale  $M_P$ , as one might naively expect, but it also has a further suppression factor proportional to

$$a_s\langle\tau_s\rangle \sim \ln(M_P/m_{3/2}) \sim \ln\mathcal{V}. \quad (3.9)$$

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<sup>4</sup>Numerically, the effect of including an uplifting potential  $\delta V \sim \frac{\epsilon}{\mathcal{V}^2}$  is to reduce  $m_\chi$  from the value given in (3.6),  $m_\chi \rightarrow 0.6m_\chi$ , while leaving  $m_\Phi$  unaffected.

<sup>5</sup>Notice that since the light field  $\chi$  is dominantly the volume modulus, for which the Kähler potential can be approximated by  $K = -3\ln(T_b + \bar{T}_b)$ . In this case one can perform the canonical normalisation for all values of the field, obtaining  $\frac{\delta\tau_b}{\tau_b} = \sqrt{\frac{2}{3}}\chi$ . This is precisely the coefficient we find in equation (3.7)

The dimensionful coupling of  $\chi$  to photons is

$$\lambda_{\chi\gamma\gamma} = \frac{\sqrt{6}}{2M_P \ln(M_P/m_{3/2})}, \quad (3.10)$$

and so it is slightly weaker than standard moduli couplings to matter. Naively one might have supposed a purely Planckian coupling, with  $\lambda_{\chi\gamma\gamma} = 1/M_P$  (as done in [15, 16, 17]). We see that the result in a more realistic model actually suppresses the decay rate by a factor of  $\ln(M_P/m_{3/2})^2 \sim 1000$ . This suppression of the  $2\gamma$  decay mode will subsequently play an important role when we discuss the possible role of  $\chi$  in generating the 511keV line from the galactic centre.

From (3.7) it also follows that the photon couplings to the heavy field  $\Phi$  will involve a factor  $\mathcal{V}^{1/2}$  rather than  $\frac{\sqrt{6}}{2a_s}$ . The dimensionful coupling is

$$\lambda_{\Phi\gamma\gamma} \sim \left( \frac{2}{\sqrt{3}} \frac{\langle\tau_b\rangle^{3/4}}{\langle\tau_s\rangle^{3/4} M_P} \right) \sim \frac{\sqrt{\mathcal{V}}}{M_P} \sim \frac{1}{m_s}. \quad (3.11)$$

This implies that the interactions of  $\Phi$  with photons are only suppressed by the string scale  $m_s \ll M_P$  rather than the Planck scale and therefore the decay rates of the heavy fields  $\Phi$  are much faster than is usually assumed for moduli fields. As we will explore later, this feature is crucial when studying the behaviour of these fields in the early universe.

### 3.2 Couplings to Electrons

Here we compute the magnitude of the modular couplings to  $e^+e^-$ . This arises from the supergravity Lagrangian, with the relevant terms being

$$\begin{aligned} \mathcal{L} &= K_{\bar{e}e} \bar{e} \gamma^\mu \partial_\mu e + K_{H\bar{H}} \partial_\mu H \partial^\mu \bar{H} + e^{\mathcal{K}/2} \partial_i \partial_j W \psi^i \psi^j, \\ &= K_{\bar{e}e} \bar{e} \gamma^\mu \partial_\mu e + K_{H\bar{H}} \partial_\mu H \partial^\mu \bar{H} + e^{\mathcal{K}/2} \lambda H \bar{e} e. \end{aligned} \quad (3.12)$$

To proceed we need to know the Kähler metric for the chiral matter fields. We use the result [25]

$$K_{\bar{e}e} \sim K_{\bar{H}H} \sim \frac{\tau_s^{1/3}}{\tau_b} = K_0 \left( 1 + \frac{1}{3} \frac{\delta\tau_s}{\langle\tau_s\rangle} - \frac{\delta\tau_b}{\langle\tau_b\rangle} + \dots \right). \quad (3.13)$$

where  $K_0 \equiv \left\langle \frac{\tau_s^{1/3}}{\tau_b} \right\rangle = \frac{\langle\tau_s\rangle^{1/3}}{\langle\tau_b\rangle}$ . We also need the expansion

$$e^{\mathcal{K}/2} = \frac{1}{\mathcal{V}} \sim \frac{9\sqrt{2}}{\tau_b^{3/2} - \tau_s^{3/2}} = \frac{1}{\mathcal{V}_0} \left( 1 - \frac{3}{2} \left( \frac{\delta\tau_b}{\langle\tau_b\rangle} \right) + \dots \right), \quad (3.14)$$

where  $\mathcal{V}_0 = \langle\mathcal{V}\rangle$ . The Lagrangian is then

$$\begin{aligned} \mathcal{L} &= K_0 \bar{e} \gamma^\mu \partial_\mu e + K_0 \partial_\mu H \partial^\mu \bar{H} + \frac{1}{\mathcal{V}_0} \lambda H \bar{e} e + \left( \frac{1}{3} \left( \frac{\delta\tau_s}{\langle\tau_s\rangle} \right) - \left( \frac{\delta\tau_b}{\langle\tau_b\rangle} \right) \right) K_0 \bar{e} \gamma^\mu \partial_\mu e \\ &\quad + \left( \frac{1}{3} \left( \frac{\delta\tau_s}{\langle\tau_s\rangle} \right) - \left( \frac{\delta\tau_b}{\langle\tau_b\rangle} \right) \right) K_0 \partial_\mu H \partial^\mu \bar{H} - \frac{3}{2} \left( \frac{\delta\tau_b}{\langle\tau_b\rangle} \right) \frac{1}{\mathcal{V}_0} \lambda H \bar{e} e. \end{aligned} \quad (3.15)$$



We can now canonically normalise the matter fields and impose electroweak symmetry breaking, giving the Higgs a vev and generating the electron mass. The effective Lagrangian for the electron field is

$$\bar{e}(\gamma^\mu \partial_\mu + m_e)e + \left(\frac{1}{3} \frac{\delta\tau_s}{\langle\tau_s\rangle} - \frac{\delta\tau_b}{\langle\tau_b\rangle}\right) \bar{e}(\gamma^\mu \partial_\mu + m_e)e - \left(\frac{1}{3} \frac{\delta\tau_s}{\langle\tau_s\rangle} + \frac{1}{2} \frac{\delta\tau_b}{\langle\tau_b\rangle}\right) m_e \bar{e}e. \quad (3.16)$$

The second term of (3.16) does not contribute to the  $\chi$  decay rate - for onshell final-state particles the Feynman amplitude vanishes due to the equations of motion. The physical decay rate is determined by the final term of (3.16),

$$\frac{1}{3} \left(\frac{\delta\tau_s}{\langle\tau_s\rangle}\right) + \frac{1}{2} \left(\frac{\delta\tau_b}{\langle\tau_b\rangle}\right), \quad (3.17)$$

and in particular how this converts into a linear combination of  $\Phi$  and  $\chi$ . Using the expression (3.7) we obtain

$$\delta\mathcal{L}_{\chi ee} \sim \left(1 + \frac{1}{a\langle\tau_s\rangle}\right) \frac{1}{\sqrt{6} M_P} \chi m_e \bar{e}e. \quad (3.18)$$

This is dominated by the former term, arising from the alignment of  $\chi$  with the overall volume direction. The coupling (3.18) is suppressed by the Planck scale, but unlike (3.8) there is no further parametric suppression.

For the heavy field  $\Phi$ , we find, similar to the couplings to photons, that the important term in (3.17) is the  $\frac{\delta\tau_s}{\langle\tau_s\rangle}$  term. Using the expansion (3.7) we again see that the coupling of  $\Phi$  to electrons is suppressed only by the string scale rather than by the Planck scale:

$$\delta\mathcal{L}_{\Phi ee} \sim \frac{\sqrt{V}\chi}{M_P} m_e \bar{e}e \sim \frac{\chi}{m_s} m_e \bar{e}e. \quad (3.19)$$

### 3.3 Computation of Moduli Lifetimes

We now use the results of the previous sections to compute the moduli lifetimes. After canonical normalisation we always obtain a Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 + \frac{\lambda\phi}{4M_P}F_{\mu\nu}F^{\mu\nu} + \mu\frac{\phi}{M_P}\bar{e}e. \quad (3.20)$$

Here  $\phi$  represents either of the fields  $\Phi, \chi$ . In terms of  $m_\phi$ ,  $\lambda$  and  $\mu$ , it is straightforward to compute the  $\phi$  decay rates, which are given by

$$\Gamma_{\phi\rightarrow\gamma\gamma} = \frac{\lambda^2 m_\phi^3}{64\pi M_P^2}, \quad (3.21)$$

$$\Gamma_{\phi\rightarrow e^+e^-} = \frac{\mu^2 m_e^2 m_\phi}{8\pi M_P^2} \left(1 - \frac{4m_e^2}{m_\phi^2}\right)^{3/2}. \quad (3.22)$$

The lifetimes for each decay mode are  $\tau = \Gamma^{-1}$ . Using  $M_P^{-1} = (2.4 \times 10^{18} \text{GeV})^{-1} = 2.7 \times 10^{-43} \text{s}$ , we can write:

$$\tau_{\phi\rightarrow\gamma\gamma} = \frac{7.5 \times 10^{23} \text{s}}{\lambda^2} \left(\frac{1 \text{MeV}}{m_\phi}\right)^3, \quad (3.23)$$

$$\tau_{\phi\rightarrow e^+e^-} = \frac{3.75 \times 10^{23} \text{s}}{\mu^2} \left(\frac{1 \text{MeV}}{m_\phi}\right) \left(1 - \left(\frac{1 \text{MeV}}{m_\phi}\right)^2\right)^{-3/2}. \quad (3.24)$$

For the light modulus  $\chi$ , substituting  $\lambda$  by  $\lambda_{\chi\gamma\gamma} \sim 1/\ln(M_P/m_{3/2}) \sim 0.038$  given in equation (3.3) and  $m_\chi \sim 2$  MeV, we have

$$\tau_{\chi \rightarrow \gamma\gamma} \sim 6 \times 10^{25} \text{ s}, \quad (3.25)$$

$$\tau_{\chi \rightarrow e^+e^-} \sim 1.7 \times 10^{24} \text{ s}, \quad (3.26)$$

which is much larger than the age of the universe  $\sim 3 \times 10^{17}$  s. From (3.24) we can see that for  $m_\chi \gtrsim 1$  MeV the decay to  $e^+e^-$  pairs is dominant, with a branching ratio  $\sim 0.97$ .

For the heavy modulus  $\Phi$ , we have  $\lambda_{\Phi\gamma\gamma} \sim \sqrt{\mathcal{V}} \sim 10^7$  and  $m_\Phi \sim 1000$  TeV. We then obtain

$$\tau_\Phi \sim 10^{-17} \text{ s}, \quad (3.27)$$

which means the heavy moduli decay very early in the history of the universe. The moduli lifetimes differ by a factor  $\sim 10^{43}$ : this large discrepancy originates in the very different masses and couplings of the two moduli.

### 3.4 Couplings and Decays to Gravitini

Another decay mode of interest is that to gravitini. This mode is interesting because of the danger of overproducing gravitini from moduli decays that give rise to reheating. While this mode is inaccessible for the light modulus  $\chi$ , for the heavy field  $\Phi$  this mode is present. In [6, 7] it was shown that for many models with heavy moduli, the gravitino branching ratio for moduli is  $\mathcal{O}(1)$ . This causes severe cosmological problems, as the decays of such gravitini either spoil nucleosynthesis or overproduce supersymmetric dark matter. However, for large volume models the branching ratio is negligible: the gravitino is a bulk mode, while the heavy modulus is located on the small cycle. While the couplings of the heavy modulus to matter are suppressed by the string scale, those to the gravitino are suppressed by the Planck scale.

For example, we can consider the  $\Phi \rightarrow 2\psi_{3/2}$  decay channel analysed in [6, 7]. This arises from the Lagrangian term

$$\begin{aligned} \mathcal{L} &\sim e^{G/2} \bar{\psi}_\mu [\gamma^\mu, \gamma^\nu] \psi_\nu \\ &= e^{G/2} \left( (\partial_{\tau_s} G) (\delta\tau_s) + (\partial_{\tau_b} G) (\delta\tau_b) \right) \bar{\psi}_\mu [\gamma^\mu, \gamma^\nu] \psi_\nu \end{aligned} \quad (3.28)$$

Here  $G = \mathcal{K} + \ln W + \ln \bar{W}$ . We now relate  $\delta\tau_s$  and  $\delta\tau_b$  to  $\Phi$  and  $\chi$  using (3.7), and use the fact that  $\partial_{\tau_s} G \sim \frac{1}{\mathcal{V}}$ ,  $\partial_{\tau_b} G \sim \frac{1}{\mathcal{V}^{2/3}}$ , to get

$$\begin{aligned} \mathcal{L} &\sim m_{3/2} \left( \frac{1}{\mathcal{V}} (\sqrt{\mathcal{V}}\Phi + \chi) + \frac{1}{\mathcal{V}^{2/3}} (\mathcal{V}^{1/6}\Phi + \mathcal{V}^{2/3}\chi) \right) \bar{\psi}_\mu [\gamma^\mu, \gamma^\nu] \psi_\nu \\ &\sim \left( \frac{1}{\sqrt{\mathcal{V}}} \frac{\Phi}{M_P} + \frac{\chi}{M_P} \right) m_{3/2} \bar{\psi}_\mu [\gamma^\mu, \gamma^\nu] \psi_\nu. \end{aligned} \quad (3.29)$$

The Lagrangian term

$$\mathcal{L} \sim \epsilon^{\mu\rho\sigma\tau} \sum \left( (\partial_{T_i} G) \partial_\rho T_i - (\partial_{\bar{T}_i} G) \partial_\rho \bar{T}_i \right) \bar{\psi}_\mu \gamma_\nu \psi_\sigma,$$

here only generates an axion-gravitino coupling and does not contribute to the  $\Phi$  decay rate. From (3.29), we then find

$$\Gamma_{\Phi \rightarrow 2\psi_{3/2}} \sim \frac{1}{\mathcal{V}} \frac{m_\Phi^3}{M_P^2}, \quad (3.30)$$

where we have focused on the dominant volume scaling. As

$$\Gamma_{\Phi \rightarrow e^+e^-} \sim \mathcal{V} \frac{m_\Phi^3}{M_P^2}, \quad (3.31)$$

(see (3.21) and (3.27) above), this implies that the branching ratio for gravitino pair production is  $\text{Br}(\Phi \rightarrow 2\psi_{3/2}) \sim \mathcal{V}^{-2} \sim 10^{-30}$ !

The striking contrast between this result and the  $\mathcal{O}(1)$  branching ratios found in [6, 7] is that for the large-volume models there exists a double suppression: first, the gravitino is a bulk mode which gives a suppression  $\left(\frac{m_s}{M_P}\right)^2 = \mathcal{V}^{-1}$ , and secondly, the dominant F-term (again by a factor of  $\mathcal{V}$ ) is that associated with the light overall volume modulus rather than the small heavy modulus.<sup>6</sup> The  $\Phi \rightarrow 2\psi_{3/2}$  decay mode is therefore suppressed by a factor  $\sim \mathcal{V}^2 \sim 10^{30}$  compared to the results of [6, 7].

In table 1 we summarise the results of this section for the properties, couplings and decay modes of the moduli. In the next sections we will examine the cosmological and astrophysical applications of these results.

	Light modulus $\chi$	Heavy Modulus $\Phi$
Mass	$\sim m_{3/2} \left(\frac{m_{3/2}}{M_P}\right)^{\frac{1}{2}} \sim 2\text{MeV}$	$2 m_{3/2} \ln(M_P/m_{3/2}) \sim 1200\text{TeV}$
Matter Couplings	$M_P^{-1}$ (electrons) $\left(M_P \ln\left(\frac{M_P}{m_{3/2}}\right)\right)^{-1}$ (photons)	$m_s^{-1}$
Decay Modes		
$\gamma\gamma$	$\text{Br} \sim 0.025, \quad \tau \sim 6.5 \times 10^{25}\text{s}$	$\text{Br} \sim \mathcal{O}(1), \quad \tau \sim 10^{-17}\text{s}$
$e^+e^-$	$\text{Br} \sim 0.975, \quad \tau \sim 1.7 \times 10^{24}\text{s}$	$\text{Br} \sim \mathcal{O}(1), \quad \tau \sim 10^{-17}\text{s}$
$q\bar{q}$	inaccessible	$\text{Br} \sim \mathcal{O}(1), \quad \tau \sim 10^{-17}\text{s}$
$\psi_{3/2}\psi_{3/2}$	inaccessible	$\text{Br} \sim 10^{-30}, \quad \tau \sim 10^{13}\text{s}$

**Table 1:** The properties of the two moduli and their decay modes. The lifetimes quoted are for sample masses of  $m_\Phi = 1200\text{TeV}$  and  $m_\chi = 2\text{MeV}$ , with a string scale of  $m_s = 10^{11}\text{GeV}$  and a gravitino mass of 20 TeV. The scale of soft terms here is  $m_{3/2}/\ln(M_P/m_{3/2}) \sim 500\text{GeV}$ .

## 4. Review of Moduli Cosmology

As mentioned in the introduction, moduli fields have been widely studied as possible candidates for inflation. There are currently several competing scenarios in which the inflaton

<sup>6</sup>We stress however that it is still the F-term  $F^\Phi$  that determines the physical soft terms, due to the much stronger matter couplings of  $\Phi$  than  $\chi$  ( $m_s^{-1}$  rather than  $M_P^{-1}$ ).

is either an open string modulus or a closed string modulus. In particular, for the large volume models there exists a natural mechanism to generate a flat potential for one of the ‘small’ Kähler moduli as long as the Calabi-Yau has more than three Kähler moduli [26]. This scenario has been further studied in [27, 28, 29, 30] where more inflationary trajectories were identified. There is a potential danger that extra quantum corrections to the Kähler potential could spoil the slow roll. However, this inflationary scenario also requires a string scale of order the GUT scale in order to achieve the correct COBE normalisation for the density perturbations. This is in tension with the scales required for particle physics, as the GUT string scale gives a very heavy gravitino  $\sim 10^{13}\text{GeV}$  (cf [31, 32]) which is incompatible with low-energy supersymmetry. This is an interesting challenge that may need realisations of inflation at a low scale [33] or a dynamical change in the volume after inflation in order to satisfy the low-energy phenomenological requirements.

For our purpose we will assume the string scale in our vacuum is the intermediate scale  $10^{11}\text{GeV}$  as preferred by particle physics. We leave as an open problem to develop a successful scenario of inflation within the context of the intermediate scale models that we consider here. Here we will simply assume that such an inflationary scenario can be developed and concentrate on the subsequent cosmological evolution after inflation, with the Kähler moduli rolling along their potential. Over the years moduli have been associated with several cosmological problems. Let us summarise the main issues.

#### 4.1 Cosmological Moduli Problem

It is well-known that generic moduli with a mass  $m \lesssim 1\text{TeV}$  pose problems for early-universe cosmology [34, 35, 36]. Such moduli masses are unavoidable in the conventional picture of gravity-mediated supersymmetry breaking, where moduli obtain masses comparable to the supersymmetry breaking scale,  $m_\phi \sim m_{3/2} \sim m_{\text{susy}}$ . In gauge-mediated models, the problem is even more serious as the moduli masses are then lower than the supersymmetry breaking scale,  $m_\phi \sim m_{3/2} \ll m_{\text{susy}}$ . The problem is that the moduli are long-lived and after inflation come to dominate the energy density of the universe.

This is a serious and model independent problem for light scalar fields that couple gravitationally. Let us briefly review the source of this problem. We assume a scalar field  $\phi$  with gravitational strength interactions in a FRW background. Its time evolution is governed by the equation<sup>7</sup>

$$\ddot{\phi} + (3H + \Gamma_\phi)\dot{\phi} + \frac{\partial V}{\partial \phi} = 0, \quad (4.1)$$

where  $H = \frac{\dot{a}}{a}$  is the Hubble parameter,  $a$  the scale factor,  $V$  the scalar potential and  $\Gamma_\phi \sim m_\phi^3/M_P^2$  the  $\phi$  decay rate. Due to its original supersymmetric flat potential, it is expected that after inflation the modulus is not at its zero-temperature minimum but instead at some initial value  $\phi_{in} \sim M_P$ . While  $t < t_{in} \sim m_\phi^{-1}$ ,  $H > m_\phi$  and the friction term  $3H\dot{\phi}$  dominates the time evolution of  $\phi$ , causing  $\phi$  to remain at  $\phi \sim \phi_{in}$ . At  $t > t_{in}$  when the universe is at a temperature  $T_{in} \sim \sqrt{m_\phi M_P}$  (since the Friedmann equation implies

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<sup>7</sup>Strictly this applies to the time-averaged amplitude of the field oscillations.

$H \sim T^2/M_P$  for radiation), the field starts oscillating around its minimum. Coherent oscillations of the field after this time will come to dominate the energy density of the universe since the initial energy density  $\rho_\phi(T_{in}) \sim m_\phi^2 \phi_{in}^2$  increases with respect to standard radiation density. The reason is that energy in coherent oscillations decreases with  $a^{-3}$  [37] whereas radiation decreases with  $a^{-4}$ . Therefore we can write:

$$\rho_\phi(T) = \rho_\phi(T_{in}) \left( \frac{T}{T_{in}} \right)^3 \sim m_\phi^2 \phi_{in}^2 \left( \frac{T_0}{\sqrt{m_\phi M_P}} \right)^3 \quad (4.2)$$

If the field  $\phi$  is stable, these oscillations will dominate the energy density of the universe and may overclose it. Imposing that  $\rho_\phi(T_0) < \rho_{critical} = 3H_0^2 M_P^2 \sim (10^{-3} \text{eV})^4$ , where  $T_0, H_0$  are the temperature and Hubble parameter today, puts a constraint on  $\phi_{in}$ ,  $\phi_{in} < 10^{-10} \left( \frac{m_\phi}{100 \text{GeV}} \right)^{-1/4} M_P$ . That is, for  $\phi_{in} \sim M_P$  a stable scalar field of mass  $m_\phi > 10^{-26} \text{eV}$  will overclose the universe.

If the scalar field decays, which is the most common situation, another problem arises. Since the field couples with gravitational strength, its decay will happen very late in the history of the universe and may spoil nucleosynthesis. This can be quantified as follows. The scalar field  $\phi$  decays at a temperature  $T_D$  for which  $H(T_D) \sim \Gamma_\phi$ . Therefore using  $\Gamma_\phi \sim m_\phi^3/M_P^2$  and the FRW equations for  $H \sim \Gamma_\phi$ :

$$\Gamma_\phi^2 \sim \left( \frac{m_\phi^3}{M_P^2} \right)^2 \sim \frac{\rho_\phi(T_D)}{M_P^2} = \frac{\rho_\phi(T_{in})}{M_P^2} \left( \frac{T_D}{T_{in}} \right)^3 \quad (4.3)$$

Using this and  $\rho_\phi(T_{in}) \sim m_\phi^2 \phi_{in}^2$ ,  $T_{in}^2 \sim m_\phi M_P$  we find the decay temperature  $T_D \sim m_\phi^{11/6} M_P^{-1/6} \phi_{in}^{-2/3}$ . At the temperature  $T_D$  the energy density  $\rho_\phi(T_D)$  gets converted into radiation of temperature

$$T_{RH} \simeq (\rho_\phi(T_D))^{1/4} \sim (M_P \Gamma_\phi)^{1/2} \sim \left( \frac{m_\phi^3}{M_P} \right)^{1/2}. \quad (4.4)$$

If  $T_{RH} \lesssim 10 \text{ MeV}$  the decay products of  $\phi$  will spoil the successful predictions of nucleosynthesis. This puts a bound on  $m_\phi$  of  $m_\phi \gtrsim 100 \text{ TeV}$ . The decay of  $\phi$  causes an increase in the entropy given by:

$$\Delta = \left( \frac{T_{RH}}{T_D} \right)^3 \sim \frac{\phi_{in}^2}{m_\phi M_P} \quad (4.5)$$

which for  $\phi_{in} \sim M_P$  gives a very large entropy increase washing out any previously generated baryon asymmetry. Therefore the standard cosmological moduli problem forbids gravity coupled scalars in the range  $m_\phi \lesssim 100 \text{ TeV}$ . We will reconsider this problem in the next subsection for the large volume string models.

## 4.2 Other Problems

- *Gravitino overproduction.* One proposal to avoid the cosmological moduli problem is through a heavy modulus scenario, where  $m_\phi \sim 1000 \text{ TeV}$  with  $m_{3/2} \sim 30 \text{ TeV}$  and  $m_{soft} \sim 1 \text{ TeV}$ . However in this case the moduli are much heavier than the gravitino

and the  $\phi \rightarrow 2\psi_{3/2}$  decay channel is open. It has recently been pointed out [6, 7] that in this case the moduli decay to gravitinos is unsuppressed and can occur with  $\mathcal{O}(1)$  branching ratio. This naturally leads to an overproduction of gravitinos at low energies, which interfere with the successful nucleosynthesis predictions. This problem appears on top of the more standard gravitino problem, in which to avoid thermal gravitino overproduction the reheating temperature must be smaller than  $10^9$  GeV.

- *Dark matter overproduction* Even in heavy moduli scenarios where the moduli mass is  $m_\phi > 100\text{TeV}$ , the reheating temperature is still very low,  $T_{\text{reheat}} \sim \mathcal{O}(10\text{MeV})$ . As the moduli mass is much greater than that of the soft terms, the moduli will also decay to TeV-scale supersymmetric particles with  $\mathcal{O}(1)$  branching ratios. The reheat temperature is much lower than that of the susy freeze-out temperature, which is typically  $T_{\text{freeze-out}} \sim m_{\text{LSP}}/20 \gtrsim \mathcal{O}(10)\text{GeV}$ . The standard thermal relic abundance computation for susy dark matter does not apply and a stable LSP is heavily overproduced.
- *Baryogenesis* Moduli decays reheat the universe, generating large amounts of entropy and diluting any primordial baryon asymmetry. At high temperatures, there exist mechanisms to generate a baryon asymmetry: for example, the electroweak sphaleron transitions that occur at  $T \sim 100\text{GeV}$  violate baryon number. However, the low reheat temperatures from moduli decay imply baryogenesis must occur at low temperatures, without the aid of the high energy baryon number-violating processes.
- *Overshooting problem.* Usually the physical minimum of the scalar potential is only a local minimum. The initial conditions may typically be that the energy is much larger than the barrier separating this minimum from the overall (zero coupling/infinite volume) minimum. The field may then roll through the local minimum and pass over the barrier. This was emphasised in reference [38]. This is a problem of initial conditions. Detailed studies of the time evolution of the scalar field, following from equation (4.1) have concluded that this problem is less severe than originally thought [39, 40]. It appears that Hubble damping together with the different redshift properties of kinetic and potential energy can be enough to avoid the field overshooting and running to infinity. This is a model dependent problem that we will not address further.
- *Inflationary destabilisation* In practical models of moduli stabilisation, the barrier height separating the true minimum from the infinite runaway is comparable to the depth of the AdS minimum, which is  $\lesssim m_{3/2}^2 M_P^2$ . The barrier height is a measure of the maximum scale at which inflation can take place, as if the inflationary energy scale is above the barrier height the potential is unstable to decompactification. During the inflationary epoch this gives a relationship  $H \lesssim m_{3/2}$  [31], which suggests that either the gravitino mass was very large during inflation  $m_{3/2} \gg 1\text{TeV}$ , or that inflation took place at a very low energy scale  $H \ll 10^{16}\text{GeV}$ . If the potential is such that the gravitino mass is  $\sim 1\text{TeV}$  during inflation, typical inflationary energy scales will destabilise the potential.

- *Temperature destabilisation.* Finite temperature effects can modify the scalar potential in such a way that the local physical minimum is washed out at finite temperature due to the  $T^4$  contribution to the scalar potential from the coupling of the modulus to a thermal matter bath. In this case the field naturally rolls towards its decompactified zero coupling limit as in the overshooting problem. If moduli fields couple to the observable sector, the free energy of a hot gas of observable particles contribute to the moduli potential since moduli correspond to gauge couplings in the effective theory. Since the free energy goes like  $T^4$ , for high enough temperatures this could destabilise the zero-temperature minimum. The critical temperature was found to be of order  $10^{13}$  GeV [41]. If inflation occurs at energies above  $10^{15}$  GeV, there is no time for observable matter to be in thermal equilibrium and the problem disappears [42]. Then for small enough reheating temperature this is not a serious problem.

## 5. Large Volume Moduli in the Early Universe

### 5.1 Cosmological Moduli Problem

Let us reanalyse the cosmological moduli problem for each of the moduli fields present in the large volume models. In total there are three classes of moduli: the complex structure and dilaton, the heavy Kähler moduli and the light Kähler modulus. Let us discuss each case on the basis of the analysis of the previous section.

1. *Complex structure and dilaton moduli.* These fields have masses of order 20 TeV and couple with gravitational strength. In principle these are in the dangerous zone for the CMP. However, as emphasised in [11], the potential for these fields dominates the overall energy density, leading to runaway behaviour, unless they sit at their minimum. The reason is that for large volumes, their contribution to the scalar potential is positive and suppressed only by  $1/\mathcal{V}^2$ , in contrast to the Kähler moduli contribution that goes like  $1/\mathcal{V}^3$  at large volume. Therefore such fields are naturally trapped at (or very close) to their minimum early in the history of the universe and are not expected to have dangerous oscillations ( $\phi_{in} \lll M_P$ ).
2. *Heavy moduli.* The heavy moduli have masses of order 1000 TeV and are coupled to matter at the string scale ( $M_s \sim 10^{11}$  GeV) rather than the Planck scale  $M_P \sim 10^{18}$  GeV). They are therefore free from the CMP as their lifetime is extremely short, with  $\tau \sim 10^{-17}$  s. Their decays will reheat the universe to

$$T_{RH} \sim (M_P \Gamma_\Phi)^{1/2} \sim (M_P m_\Phi / M_s^2)^{1/2} m_\Phi \sim 10^7 \text{ GeV}. \quad (5.1)$$

Furthermore, as the couplings of these moduli to the gravitini are Planck suppressed rather than string suppressed, gravitino decay modes have tiny branching ratios. For example, the  $\Phi \rightarrow 2\psi_{3/2}$  decay mode occurs with a branching ratio of  $\sim 10^{-30}$ , in contrast to the  $\mathcal{O}(1)$  expectations of [6, 7].

As the reheat temperature is high, it is possible to start a Hot Big Bang at a relatively high temperature, with the possibility of a conventional treatment of susy decoupling

and axion evolution. For the above reasons such moduli are very attractive for reheating the universe after inflation.

3. *Light modulus.* This field has a mass of order 1 MeV with gravitational strength interactions and it is thus dangerous for the CMP. Notice that standard inflation can never address the CMP because there is no reason for the scalar field to be at its minimum just after inflation. To solve this problem we need to have either a trapping mechanism to keep the fields in or close to their minima or alternatively a period of late inflation. The best option for this is thermal inflation [43] that we will discuss next.

## 5.2 Thermal Inflation

Thermal inflation is not just another particular choice of scalar field and potential energy to give rise to slow-roll inflation at high energies. Thermal inflation is rather a general class of models that tend to induce a short period of low-temperature inflation in a natural way. It is not an alternative to slow-roll inflation to solve the big-bang problems and produce the density perturbations, but instead complements it with a short period of low energy inflation that can dilute some relic particles.

Thermal inflation was proposed in [43]. The observation is that in supersymmetric models there are many flat directions (such as the string moduli and others) that are lifted after supersymmetry breaking. A field with such a flat direction, which we denote by  $\sigma$ , can have a vacuum expectation value (*vev*) much larger than its mass. If this is the case  $\sigma$  is called a ‘flaton’ field (not to be confused with the inflaton).

The cosmological implications of a flaton field are quite interesting. If the flaton field is in thermal equilibrium with matter, there is a finite temperature contribution to its scalar potential:

$$V = V_0 + (T^2 - m_\sigma^2) \sigma^2 + \dots \quad (5.2)$$

where we have expanded around a local maximum of  $\sigma$  taken to be at  $\langle \sigma \rangle = 0$ . This is a false vacuum at temperatures  $T > T_c = m_\sigma$ . At these temperatures  $\sigma$  will be trapped at the origin. The zero temperature minimum is at  $\langle \sigma \rangle \equiv M_* \gg m_\sigma$ . At a particular temperature  $T \simeq V_0^{1/4} > T_c$ , the potential energy density  $V_0$  starts to dominate over the radiation energy  $\sim T^4$  and a short period of inflation develops. Inflation ends at  $T = T_c$  when the field  $\sigma$  becomes tachyonic at the origin and runs towards its zero temperature minimum. The number of efolds during this period of inflation is

$$N \sim \log \left( V_0^{1/4} / T_c \right) \sim \log (M_* / m_\sigma)^{1/2},$$

where we have used that during inflation the scale factor is inversely proportional to the temperature and  $V_0 \simeq M_*^2 m_\sigma^2$ . For  $m_\sigma \sim 1$  TeV and  $M_* \sim 10^{11}$  GeV, the number of  $e$ -folds is  $N \sim 10$ . This is large enough to dilute the surviving moduli and solve the cosmological moduli problem, but small enough to not interfere substantially with the density perturbations coming from the original period of inflation at higher energies.

It is interesting that the values preferred for the scales  $M_*$  and  $m_\sigma$  are precisely the string and soft SUSY breaking scales in our scenario. It therefore seems natural to try to



implement thermal inflation in this scenario with  $M_* = M_s, m_\sigma \sim m_{3/2}$ . Candidate flaton fields can be any moduli with  $vev$  of order one in string units and masses of the order of the soft masses. Singlet open string modes abound in D-brane constructions that have precisely these properties. The heavy Kähler moduli also have the right mass scale and  $vev$ . However their coupling to matter is suppressed by the string scale and it is difficult for them to be in thermal equilibrium with observable matter.<sup>8</sup> An explicit realisation of thermal inflation in our class of models is beyond the scope of the present article, but it is encouraging to see that they do have the right properties for thermal inflation to happen with several candidate flaton fields. There is actually an explicit candidate for thermal inflation using the properties of D-branes [44].

Other scenarios of low-temperature inflation could also work. Although standard slow-roll inflation is difficult to obtain at low energies, other variants such as locked inflation [45] could be promising, especially if they could be implemented within string theory. A period of low-temperature inflation has also been proposed in recent attempts to derive inflation from string theory [46].

### 5.3 Comparison with Other Scenarios

Even though the moduli are generic in string compactifications their physical implications change considerably depending on the details of moduli stabilisation and supersymmetry breaking. At the moment there are at least four main scenarios that can be distinguished:

1. The generic gravity mediated scenario. In this case all moduli are expected to get a mass proportional to the gravitino mass. The argument is that their mass has to be proportional to the auxiliary field that breaks supersymmetry divided by the strength of the interaction that mediates the breaking of supersymmetry ( $m_\phi \sim \langle F \rangle / M_P$ ) which is precisely the gravitino mass  $m_{3/2} \sim 1$  TeV. All moduli are assumed to couple with gravitational strength, and all moduli suffer from the cosmological moduli problem.
2. Generic gauge mediated supersymmetry breaking. In this case  $m_{3/2} \ll 1$  TeV. The moduli masses are still of the same order of the gravitino mass, but now this may be as low as  $m_{3/2} \sim \langle F \rangle / M_P \sim 10^{-3} - 10^3$  eV. They also couple with gravitational strength and induce a CMP even more severe than for the gravity-mediated scenario.
3. Mirage mediation [47]. This differs from conventional gravity mediation in the fact that the moduli masses are  $m_\phi \sim m_{3/2} \log(M_P / m_{3/2}) \sim 1000$  TeV. This improves on the CMP as moduli decay prior to BBN, but gives new problems with the overproduction of gravitini and susy dark matter as discussed above.

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<sup>8</sup>This suppression is also present for the typical flaton fields considered in the literature. To be in thermal equilibrium with matter it is usually assumed that the flaton field couples to massive particles with a mass given by  $\langle \sigma \rangle$ . When  $\langle \sigma \rangle$  is close to zero these fields are light and allow  $\sigma$  to be in thermal equilibrium. We may envisage a similar situation for the heavy moduli, as their vanishing implies a four-cycle collapsing and the appearance of extra massless fields. A proper treatment of this interesting is beyond the low-energy effective action we have been using and would require further study.

4. Large volume models. In our case there are different classes of moduli. The heavy moduli with  $m_\Phi \sim 2m_{3/2} \log(M_P/m_{3/2}) \sim 1000$  TeV are free from both the CMP and gravitino overproduction problems because their couplings are only suppressed by the string scale. The light volume modulus has mass  $\sim 1$  MeV and couples gravitationally, and is subject to the CMP.

The moduli spectrum for large-volume models does not remove all cosmological problems. However, it does give quite different behaviour to more standard expectations. One striking difference is the possibility of a high moduli reheating temperature,  $T_{RH} \sim 10^7$  GeV, and the commencement of a Hot Big Bang at a relatively early stage. This arises because there exist moduli coupled to matter at the string, rather than the Planck, scale. In the standard case where all moduli couple to matter at the Planck scale, the reheating temperature is invariably low. For TeV scale moduli,  $T_{RH} < 1$  MeV and nucleosynthesis fails. Even in scenarios with heavy moduli, with  $m_\Phi \sim 1000$  TeV, the reheating temperature is still  $T_{RH} < 1$  GeV. High reheating temperatures are attractive because they can provide the necessary initial conditions for a period of thermal inflation or for the standard susy relic abundance computation.

The other striking difference in the spectrum of the large-volume models is the volume modulus. This is extremely light ( $\sim 1$  MeV) and gravitationally coupled; such a field is unusual in models of gravity-mediated supersymmetry breaking. Even if a Hot Big Bang has started at  $10^7$  GeV, this field will subsequently come to dominate the energy density of the universe if its abundance is not diluted. This is why a period of late-time (thermal) inflation may be necessary in order to dilute this volume modulus. We now investigate the properties of this field in more detail.

## 6. Large Volume Moduli in the Late Universe

The combination of a light  $\mathcal{O}(\text{MeV})$  modulus with gravity-mediated TeV-scale supersymmetry breaking is an unusual and distinctive feature of the large-volume models, and offers the chance of obtaining a smoking-gun signal for this class of models. As the volume modulus is stable on the lifetime of the universe, it may be present today as part of the dark matter. As analysed in section 3 above, it is unstable and may decay to  $\gamma\gamma$  or, if kinematically accessible,  $e^+e^-$ .

We here analyse the possibilities for detecting these decays. We first consider the photon flux due to  $\chi \rightarrow \gamma\gamma$  decays, considering several astrophysical sources. In section 6.2 we generalise this to include the dominant decay mode  $\chi \rightarrow e^+e^-$ , and discuss the relevance of this decay to the 511 keV positron annihilation line from the galactic centre. We start by leaving the lifetime,  $\tau_\chi$ , and mass,  $m_\chi$  of the modulus unspecified: these will subsequently be set as in section 3 above.

### 6.1 Photon flux from $\chi \rightarrow \gamma\gamma$ decays

As sources, we consider the Milky Way halo, the diffuse background and nearby galaxy clusters. We assume the field  $\chi$  constitutes a fraction  $\Omega_\chi/\Omega_{dm}$  of the dark matter.

## The Milky Way Halo

We assume the Milky Way halo to be spherical. For definiteness we consider two dark matter profiles, isothermal and Navarro-Frenk-White (NFW), as these both allow an analytic treatment. These are

$$\rho_I(r) = \frac{\rho_0}{1 + \frac{r^2}{r_c^2}}, \quad \rho_{NFW}(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right) \left(1 + \frac{r}{r_s}\right)^2}. \quad (6.1)$$

For both halo models,  $\rho_0$ ,  $r_c$  and  $r_s$  are phenomenological parameters.  $r$  is measured from the galactic centre. By relating galactic coordinates  $(x, b, l)$  to Cartesian coordinates on the galactic centre, we can write

$$\begin{aligned} r^2 &= (-R_0 + x \cos b \cos l)^2 + (x \cos b \sin l)^2 + (x \sin b)^2 \\ &= (x - R_0 \cos b \cos l)^2 + R_0^2(1 - \cos^2 b \cos^2 l). \end{aligned} \quad (6.2)$$

Here  $R_0 \sim 8\text{kpc}$  is the distance of the sun from the galactic centre. (6.2) allows the computation of  $\rho(x, b, l)$  for any given halo model.

If the decay  $\chi \rightarrow \gamma\gamma$  occurs at distance  $x$  from a detector with cross-section  $\Delta_D$ , the probability that a photon reaches the detector is  $\mathcal{P} = \frac{\Delta_D}{4\pi x^2} \times 2$ , where the factor of 2 accounts for the two photons from the decay. The number of photons arriving from distances between  $x$  and  $x + dx$  in time  $dt$  within a solid angle  $d\Sigma$  is

$$\underbrace{\frac{dt}{\tau_\chi}}_{\text{fractional decay probability}} \times \underbrace{n(\chi, x) \times (x^2 dx) \times d\Sigma}_{\text{no. of particles}} \times \underbrace{\frac{\Delta_D}{4\pi x^2} \times 2}_{\text{arriving photons per decay}}. \quad (6.3)$$

To obtain the total number of arriving photons, we integrate this quantity along the radial ( $x$ ) direction, to obtain

$$\mathcal{N}_\gamma(b, l) = \Delta_D \times dt \times \frac{2}{\tau_\chi m_\chi} \times \frac{d\Sigma}{4\pi} \times \left( \frac{\Omega_\chi}{\Omega_{dm}} \right) \int dx \rho(x). \quad (6.4)$$

We now perform the  $\int dx \rho(x)$  integral for both the profiles considered.

### 1. Isothermal Profile

Here

$$\rho_I(x) = \frac{\rho_0 r_c^2}{r_c^2 + (x - R_0 \cos b \cos l)^2 + R_0^2(1 - \cos^2 b \cos^2 l)}. \quad (6.5)$$

Defining  $R_{eff}^2 = r_c^2 + R_0^2(1 - \cos^2 b \cos^2 l)$ , we can do the integral using standard trigonometric substitutions, obtaining for the number of photons arriving per unit time

$$N_\gamma = (\Delta_D) dt \left( \frac{d\Sigma}{4\pi} \right) \frac{2}{\tau_\chi m_\chi} \left( \frac{\Omega_\chi}{\Omega_{dm}} \right) \rho_0 r_c^2 \left[ \frac{1}{R_{eff}} \left( \frac{\pi}{2} + \arctan \left( \frac{R_0 \cos b \cos l}{R_{eff}} \right) \right) \right] \quad (6.6)$$

These photons are all mono-energetic of energy  $\frac{m_\chi}{2}$  and will appear as a monochromatic line of width  $\Delta E$ , the energy resolution of the detector at  $E \sim \frac{m_\chi}{2}$ . The intensity of this line is

$$I_{line}(b, l) = \frac{N_\gamma(b, l)}{\Delta E}. \quad (6.7)$$

## 2. Navarro-Frenk-White Profile

For this case the integral  $\int dx \rho(x)$  is performed in the appendix. The resulting number density of arriving photons is given by

$$N_\gamma(b, l) = \Delta_D dt \left( \frac{d\Sigma}{4\pi} \right) \frac{2}{\tau_\chi m_\chi} \left( \frac{\Omega_\chi}{\Omega_{dm}} \right) \rho_0 r_s^3 X(b, l), \quad (6.8)$$

where

$$\begin{aligned} X(b, l) \equiv & \frac{1}{r_s^2 - R_1^2(b, l)} \left( -1 - \frac{R_0^2 - R_1^2(b, l)}{R_0 + r_s} \right) \\ & - \frac{r_s}{(r_s^2 - R_1^2(b, l))^{3/2}} \ln \left[ \frac{r_s R_0 + R_1^2(b, l) - \sqrt{(r_s^2 - R_1^2(b, l))(R_0^2 - R_1^2(b, l))}}{R_1(b, l)(r_s + R_0)} \right] \\ & + \frac{r_s}{(r_s^2 - R_1^2(b, l))^{3/2}} \ln \left[ \frac{R_1(b, l)}{r_s - \sqrt{r_s^2 - R_1^2(b, l)}} \right], \end{aligned} \quad (6.9)$$

with  $R_1(b, l) = \sqrt{R_0^2(1 - \cos^2 b \cos^2 l)}$ . As before,

$$I_\gamma(b, l) = \frac{N_\gamma}{\Delta E}. \quad (6.10)$$

For numerical evaluations we use for the isothermal profile  $\rho_0 = 7.8 \text{ GeV cm}^{-3}$  and  $r_c = 2 \text{ kpc}$ , whereas for the NFW profile we use [48]  $\rho_0 = 0.23 \text{ GeV cm}^{-3}$ ,  $r_s = 27 \text{ kpc}$ , in both cases corresponding to  $\rho(R_0) = 0.46 \text{ GeV cm}^{-3}$ .

The galactic centre region, near  $(b, l) = (0, 0)$  is one of the most intensively observed areas of the galaxy and should contain an excess of dark matter. It should therefore provide the best sensitivity in a search for a gamma-ray line due to modulus decay. Integrating over a region  $-15^\circ < b < 15^\circ, -15^\circ < l < 15^\circ$  for an NFW profile, we find a total photon flux of

$$\mathcal{N}_\gamma = \left( \frac{\Omega_\chi}{\Omega_{dm}} \right) \left( \frac{6.5 \times 10^{25} s}{\tau_{\chi \rightarrow \gamma\gamma}} \right) \left( \frac{2 \text{ MeV}}{m_\chi} \right) \times (2.9 \times 10^{-2} \text{ photons cm}^{-2} \text{ s}^{-1}). \quad (6.11)$$

The isothermal profile gives similar results. The INTEGRAL upper bound on  $\sim 1 \text{ MeV}$  gamma-ray lines from the galactic centre is that the line strength be  $\lesssim 5 \times 10^{-5} \text{ photons cm}^{-2} \text{ s}^{-1}$  [49, 50], so the absence of any such line constrains

$$\frac{\Omega_\chi}{\Omega_{dm}} \lesssim 10^{-3} \left( \frac{2 \text{ MeV}}{m_\chi} \right)^2. \quad (6.12)$$

## Diffuse Background Emission

Moduli decays across the history of the universe also contribute to the diffuse photon background. We again relegate the computational details to the appendix, where we show that the resulting photon flux intensity is

$$I_\gamma(E) = \frac{d\Sigma}{4\pi} \times \Delta_D \times dt \times dE_\gamma \times \left(\frac{\Omega_\chi}{\Omega_m}\right) \frac{2\rho_0}{\tau_\chi m_\chi} E_\gamma^{\frac{1}{2}} \left(\frac{2}{m_\chi}\right)^{3/2} f\left(\frac{E'}{E_\gamma}\right) \frac{c}{H_0}, \quad (6.13)$$

with  $c$  the speed of light and

$$f(x) = \left[ \Omega_m + \frac{1 - \Omega_m - \Omega_\Lambda}{x} + \frac{\Omega_\Lambda}{x^3} \right]^{-\frac{1}{2}}.$$

$\tau_\chi$  is the modulus lifetime and  $\rho_0$  the current dark matter density.  $E' \equiv \frac{m_\chi}{2}$  is the original decay energy of the photons.

Because of the assumptions of homogeneity and isotropy, this quantity will have the same value irrespective of direction. In figure 1, we plot this quantity together with a fit to the extragalactic diffuse gamma-ray background observed by COMPTEL. For  $800\text{keV} < E_\gamma < 30\text{MeV}$  this is fit by [51]

$$I_\gamma(E) = \left(\frac{E}{5\text{MeV}}\right)^{-2.4} \times (1.05 \times 10^{-4} \text{ photons cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\text{MeV}^{-1}). \quad (6.14)$$

We see that for  $m_\chi \gtrsim 1\text{MeV}$  the combination of (6.13) and (6.14) constrains the allowed  $\chi$  density to be

$$\frac{\Omega_\chi}{\Omega_m} \lesssim \left(\frac{1\text{MeV}}{m_\chi}\right)^{3.5}. \quad (6.15)$$

## Galaxy Clusters

We can also consider specific local galaxy clusters. A galaxy cluster is a locally overdense region of the sky, at a specific distance  $D$  from the earth. We denote the total dark mass of the cluster by  $M$ , with a fraction  $\Omega_\chi/\Omega_m$  consisting of moduli. The total number of moduli is then  $\left(\frac{M}{m_\chi}\right) \left(\frac{\Omega_\chi}{\Omega_m}\right)$ , and thus the total number of arriving photons is

$$\frac{\Delta_D}{4\pi D^2} \times 2 \times \left(\frac{M}{m_\chi \tau_\chi}\right) \left(\frac{\Omega_\chi}{\Omega_m}\right).$$

The photons give a monochromatic line of intensity

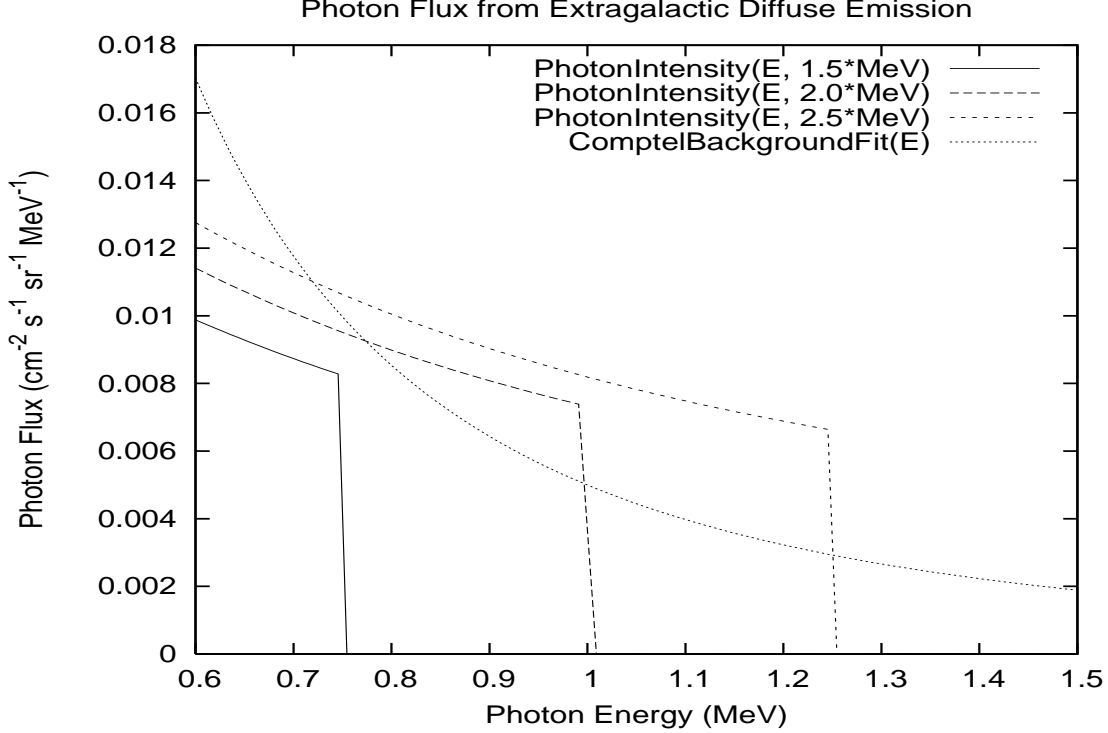
$$I_\gamma = \frac{\Delta_D}{4\pi D^2} \times 2 \times \left(\frac{M}{m_\chi \tau_\chi}\right) \left(\frac{\Omega_\chi}{\Omega_m}\right).$$

The line is redshifted from  $E = \frac{m_\chi}{2}$  according to the distance of the cluster. Considering for example the Coma or Perseus galaxy clusters, we find

$$I_{cluster} \sim 5 \times 10^{-6} \text{ photons cm}^{-2}\text{s}^{-1} \left(\frac{1.5\text{MeV}}{m_\chi}\right) \left(\frac{1.4 \times 10^{26}\text{s}}{\tau_{\chi \rightarrow \gamma\gamma}}\right) \left(\frac{\Omega_\chi}{\Omega_m}\right) \quad (6.16)$$

$$= 5 \times 10^{-6} \text{ photons cm}^{-2}\text{s}^{-1} \left(\frac{1.5\text{MeV}}{m_\chi}\right)^2 \left(\frac{\Omega_\chi}{\Omega_m}\right). \quad (6.17)$$

This does not provide a competitive constraint on the modulus density.



**Figure 1:** The extragalactic diffuse photon flux arising from moduli decays through the history of the universe. We plot the flux arising for  $\left(\frac{\Omega_\chi}{\Omega_{dm}}\right) = 1$  for moduli masses  $m_\chi = 1.5, 2$  and  $2.5$  MeV. We use the results of (3.10) for the coupling of  $\chi$  to photons. As comparison we also plot a fit to the extragalactic diffuse gamma-ray background observed by COMPTEL.

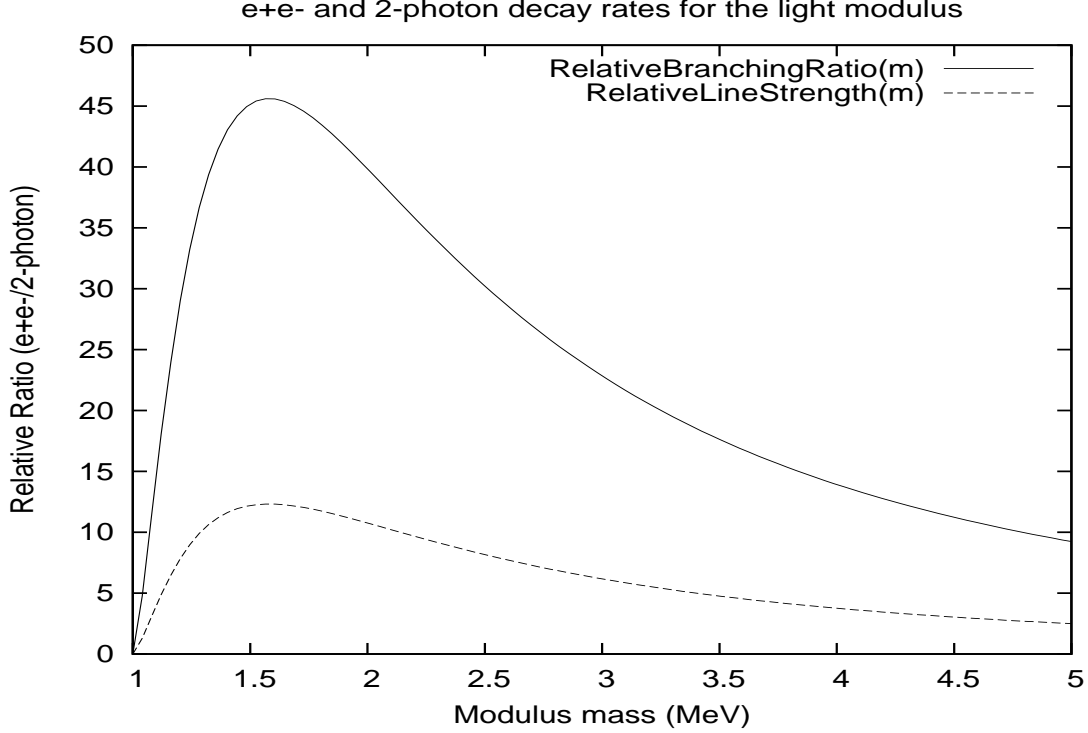
## 6.2 $\chi \rightarrow e^+e^-$ decays and the 511keV line

If kinematically accessible, the light modulus  $\chi$  can also decay to  $e^+e^-$  pairs with a decay rate

$$\Gamma_{\chi \rightarrow e^+e^-} = \frac{m_e^2 m_\chi}{48\pi M_P^2} \left(1 - \frac{4m_e^2}{m_\chi^2}\right)^{3/2}. \quad (6.18)$$

For masses  $m_\chi \gtrsim 1$  MeV, this is the dominant decay mode. A plot of the relative branching fractions for  $\gamma\gamma$  and  $e^+e^-$  are shown in figure 2. The decays of such a modulus could then be observed through the positrons produced in the decay.

There exists an excess of 511keV photons from the galactic centre, which has been detected for many years [52, 53]. On the earth the flux of 511keV photons is measured by the SPI spectrometer on the INTEGRAL satellite to be  $(0.96 \pm 0.06) \times 10^{-3}$  photons  $\text{cm}^{-2} \text{s}^{-1}$  [54] This flux originates from the Milky Way bulge and it appears difficult for traditional astrophysical sources to account for the intensity and location of the line. The origin and injection energy of the positrons is unknown. Positrons are produced relativistically and lose energy through ionisation and diffusion processes. Eventually they become non-relativistic, annihilating with electrons to produce gamma rays. The strongest constraint on the injection energy of the positrons comes from inflight annihilation of energetic positrons to generate diffuse gamma rays above 511 keV. The lack of an excess in the diffuse gamma



**Figure 2:** The relative branching ratio  $Br(\chi \rightarrow e^+e^-)/Br(\chi \rightarrow 2\gamma)$  and line strengths arising from  $\chi \rightarrow e^+e^-$  and  $\chi \rightarrow 2\gamma$  decay modes. The  $e^+e^-$  decay mode dominates within the interesting range of moduli masses. For the  $\chi \rightarrow e^+e^-$  decay mode, ‘line strength’ corresponds to that of the 511keV line from positron annihilation, taking into account  $3\gamma$  positronium decay.

ray spectrum requires the positron injection energy to be  $\lesssim 3\text{MeV}$  [55] (more conservative models may extend this to  $\lesssim 10\text{MeV}$  [56]). The default assumption has to be that the positron excess arises from incompletely understood astrophysics associated with e.g. the old stellar population of the galaxy. Nonetheless there remains the exciting possibility that the excess has an exotic origin, such as arising from annihilating or decaying dark matter [57, 58, 59].

In [60] decaying string moduli were considered as possible candidates to explain the 511keV line. However, due to the above constraints on injection energies,  $E_{inj} \lesssim 3(10)\text{MeV}$ , the modulus mass can only lie in a narrow window  $1\text{MeV} \lesssim m_\chi \lesssim 6\text{MeV}(20\text{MeV})$ . Furthermore, the  $2\gamma$  decay must be considerably suppressed to avoid a photon line dramatically exceeding that of the 511 keV line. In [60] it was necessary to impose both of the above constraints by hand. In this respect the volume modulus  $\chi$  that arises in the large-volume models is very appealing. The mass scale arises in exactly the regime required,  $\sim 1\text{MeV}$ , to avoid an overly high positron injection energy. This mass scale is closely tied to the existence of TeV-scale supersymmetry and thus has a limited range of allowed variation. The  $2\gamma$  decay rate also has a substantial parametric suppression in the coupling, by a factor  $(\ln(M_P/m_{3/2}))^2$ , compared to the  $e^+e^-$  decay mode. Such a large suppression is essential: both phase space effects and the  $3\gamma$  positronium decay make  $e^+e^-$  decays less efficient at

generating photons than the direct  $2\gamma$  decay.

For the decays of the light modulus to saturate the 511keV line, we would require  $\Omega_\chi \sim (\text{a few}) \times 10^{-4}$ , depending on the details of the dark matter profile. This follows from an identical computation to (6.11), except using the  $\chi \rightarrow e^+e^-$  decay rather than the  $\chi \rightarrow \gamma\gamma$  decay model.<sup>9</sup> We can then use the computed couplings of the light modulus  $\chi$  to electrons and photons to work out the relative intensity of the 511 keV line from  $\chi \rightarrow e^+e^- \rightarrow \dots \rightarrow e^+e^- \rightarrow \gamma\gamma$  and the monochromatic photon line arising from the direct decay of the modulus  $\chi \rightarrow 2\gamma$ .<sup>10</sup> The strength of the 511keV line depends on the fraction  $f$  of  $e^+e^-$  pairs that first form positronium, which goes into  $2\gamma$  and  $3\gamma$  final states with branching ratios 0.25 : 0.75. Only the  $2\gamma$  decay generates 511keV line emission whereas the  $3\gamma$  decay gives continuum emission. Assuming all positrons annihilate non-relativistically, the relative magnitudes of the  $3\gamma$  positronium continuum and  $2\gamma$  direct annihilation fix the fraction of positrons that annihilate via positronium to be  $f = 0.97 \pm 0.02$  [62]. The number of photons produced per  $\chi \rightarrow e^+e^-$  decay is then  $2((1-f) + 0.25f) = 0.54$ , in contrast to the 2 photons produced per  $\chi \rightarrow \gamma\gamma$  decay. The relative intensities of the 511keV line and the line from direct  $2\gamma$  decay is then

$$\mathcal{R} = \frac{Br(\chi \rightarrow e^+e^-)}{Br(\chi \rightarrow \gamma\gamma)} \times \frac{0.54}{2}. \quad (6.19)$$

For the  $\mathbb{P}_{[1,1,1,6,9]}^4$  model analysed in detail in this paper, we plot  $\mathcal{R}$  as a function of  $m_\chi$  in figure 2. We see that  $\mathcal{R} \lesssim 12$ , which would correspond to a line intensity from direct decay of  $\sim 8 \times 10^{-5} \text{photons cm}^{-2}\text{s}^{-1}$ . INTEGRAL constrains the strength of new gamma-ray lines of  $E_\gamma \sim 1\text{MeV}$  from the galactic center to be  $\lesssim 5 \times 10^{-5} \text{photons cm}^{-2}\text{s}^{-1}$  [49, 50], and so the existence of such a line is marginally ruled out.

There are however two caveats. The first is that the couplings have been computed in the high-energy theory, and so are valid at the scale  $m_s \sim 10^{11}\text{GeV}$ . These should properly be renormalised down to the scale  $m_\chi \sim 1\text{MeV}$ . By analogy with the running of gauge couplings, or soft masses in supersymmetric scenarios, this may introduce  $\mathcal{O}(1)$  corrections to the high-scale coupling constants given in (3.10) and (3.18). The precise results of the renormalisation depends on the details of the charged matter at  $E \gtrsim 1\text{TeV}$ . However, we can see that such corrections could easily reduce the strength of the  $\gamma\gamma$  line below the observable limit.

The second caveat is that while the coupling (3.18) to electrons is model-independent - i.e. independent of the choice of Calabi-Yau geometry and the details of the moduli stabilisation - the coupling (3.10) to photons is not and is specific to the  $\mathbb{P}_{[1,1,1,6,9]}^4$  model considered here. This is because the derivation of the coupling to electrons depended only on the powers of volume present in the matter kinetic terms. This volume scaling can

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<sup>9</sup>It is argued in [61] that for decaying dark matter to generate the 511keV line a very cuspy dark matter profile would be necessary. However there are substantial astrophysical uncertainties on the dark matter profile at small scales, and we also note that as  $\Omega_\chi/\Omega_{dm} \lesssim 10^{-3}$  the  $\chi$  profile need not precisely coincide with the galactic dark matter profile.

<sup>10</sup>This contains the implicit assumption that the locus of positron annihilations and the locus of modulus decays are approximately the same.



be determined independently of any assumptions about the geometry of the small cycle and precisely where the Standard Model is realised. In contrast, the coupling to photons depends on the extent to which the light  $\chi$  modulus has a component along the small cycle on which the Standard Model is supported. This is more dependent on the details of the moduli stabilisation and on the geometry used to realise the Standard Model.

As the exclusion is only marginal it therefore remains possible that the  $e^+e^-$  decays of the volume modulus could saturate the 511keV flux while the  $2\gamma$  decays are below the currently observable limit. Clearly any future observation of a new gamma ray photon line from the galactic centre would greatly clarify this issue.

We also note that as the mass of the volume modulus is  $\sim 1\text{MeV}$  it is certainly possible that  $m_\chi < 1\text{MeV}$ . In this case the  $e^+e^-$  decay is not kinematically accessible and there is no possibility of accounting for the 511keV line through  $\chi$  decays.

## 7. Conclusions

In this paper we have analysed the spectrum, couplings, decay modes and branching ratios of the moduli for large-volume IIB string models. The supersymmetry breaking moduli divide into two classes with quite distinct properties.

1. The first class consists of heavy moduli, with a mass  $m_\Phi \sim 2\ln(M_P/m_{3/2})m_{3/2}$ . For a realistic supersymmetry breaking scale  $m_{\text{soft}} \sim m_{3/2}/\ln(M_P/m_{3/2}) \sim 1\text{TeV}$ , this mass is  $\sim 1000\text{TeV}$ . These moduli are coupled to matter at the string scale ( $\sim 10^{11}\text{GeV}$ ) rather than the Planck scale ( $\sim 10^{18}\text{GeV}$ ). They decay very rapidly, with a lifetime  $\tau \sim 10^{-17}\text{s}$ , reheating to a temperature  $\sim 10^7\text{GeV}$ . The couplings of such moduli to gravitini are Planck-suppressed rather than string-suppressed. The heavy modulus decay  $\Phi \rightarrow \psi_{3/2}\psi_{3/2}$  has a branching ratio  $\sim 10^{-30}$ , and these moduli suffer neither from the cosmological moduli problem nor the gravitino overproduction problem.
2. The second class consists of the modulus controlling the overall volume. This has a mass  $m \sim m_{3/2} \left(\frac{m_{3/2}}{M_P}\right)^{\frac{1}{2}}$ . With TeV-supersymmetry breaking, this corresponds to  $m_\chi \sim 1\text{MeV}$ . This field couples to matter at the Planck scale rather than the string scale, and is stable on the lifetime of the universe, with a decay lifetime  $\tau \sim 10^{26}\text{s}$ . The existence of a light  $\sim 1\text{MeV}$  gravitationally coupled scalar is an extremely robust and model-independent prediction of the large-volume scenario.

The spectrum and properties of these moduli fields are quite distinct to those encountered in other scenarios of supersymmetry breaking. In particular, the combination of a light 1MeV modulus with gravity mediated supersymmetry breaking is a very distinctive prediction of the large-volume models.

While the analysis focused on one particular large-volume model, we anticipate that this division will be more general, with many small heavy moduli  $\Phi_i$  and one light volume modulus  $\chi$ .<sup>11</sup> The overall volume dependence of the couplings are expected to be general

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<sup>11</sup>For fibrations, there may also exist additional light moduli with  $m \sim m_{3/2} \left(\frac{m_{3/2}}{M_P}\right)^{2/3}$  [64].

whereas the numerical coefficients will vary from model to model, depending on the geometry of the corresponding Calabi-Yau manifold. We also concentrated on the coupling of the moduli to QED since electrons, positrons and photons are the only allowed decay channels for the light modulus  $\chi$  (neutrino interactions are suppressed by  $m_\nu$ ). The coupling of  $\chi$  to QCD and baryonic matter can still be interesting to put constraints on the effects  $\chi$  may have at low energies. This can be done on the same lines as [63], where a careful analysis was made for all masses of dilaton-like particles like  $\chi$ . The range of masses of  $\chi$  is too low to allow decays into baryonic matter and too large to be severely constrained by violations of the equivalence principle.

We analysed the effect of the above moduli spectrum on early-universe cosmology. While this moduli spectrum does not solve all cosmological problems, it does provide a quite different cosmological evolution. As the heavy moduli are coupled to matter at the string scale, they decay very rapidly, creating an early Hot Big Bang with a temperature  $T \sim 10^7 \text{ GeV}$ . The reheating associated with these moduli does not come with a gravitino overproduction problem, as the gravitino is only Planck-scale coupled whereas ordinary matter is string-scale coupled. However, the light modulus is subject to the cosmological moduli problem and will tend to overclose the universe unless its abundance is diluted. To dilute this abundance a period of thermal or alternative low-temperature inflation would seem necessary; it may be possible to achieve this using the high temperatures generated by the decay of the heavy moduli.

We also analysed the astrophysical consequences of the light modulus, assuming it to be present today as part of the dark matter. If this is the case this field may be detected through its decays to  $2\gamma$  or  $e^+e^-$ . We computed its couplings and showed that its branching ratio to photons is parametrically suppressed by a factor  $\ln(M_P/m_{3/2})^2$  compared to the branching ratio to  $e^+e^-$ . As the  $e^+e^-$  branching ratio is dominant, this opens the intriguing possibility that the decays of this field are responsible for the 511keV line observed from the galactic centre. In this regard the scale of the light modulus at  $\mathcal{O}(1)\text{MeV}$  is attractive, as the positron injection energies are required from astrophysics to be  $\lesssim 3\text{MeV}$ . Using the tree-level high-scale couplings for the  $\mathbb{P}^4_{[1,1,1,6,9]}$  model, this possibility is marginally excluded, as we showed the  $2\gamma$  decay line would then be slightly stronger than the observational bounds. However, these couplings will be affected both by renormalisation to the low scale and by differing numerical coefficients that depend on the detailed geometry of the precise Calabi-Yau used, and so the possibility that the decays of the light modulus is responsible for the 511 keV line therefore remains open.

Finally, due to the  $(\ln(M_P/m_{3/2}))^2$  magnitude of the suppression for the monochromatic  $2\gamma$  decay line, it may be argued that if the 511 keV line is due to the  $e^+e^-$  decay of the light modulus, the monochromatic  $2\gamma$  line should be within reach in the near future. It is also interesting to notice that the lifetime of the light modulus is in the range that is close to being constrained by current CMB data [65] and future observations of the Hydrogen 21 cm line [66].

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## A. Moduli Kinetic Terms and Mass Matrices

The kinetic terms for the moduli fields can be computed from the Kähler potential. As we work at large volume, in computing the kinetic terms we drop the  $\alpha'$  corrections and other terms that are suppressed at large volume. Keeping the contributions to the metric at leading order in  $\tau_b$ , we have

$$\mathcal{K} = -2 \ln \left( \frac{1}{9\sqrt{2}} \left( \tau_b^{3/2} - \tau_s^{3/2} \right) + \frac{\xi}{2g_s^{3/2}} \right), \quad (\text{A.1})$$

giving

$$\mathcal{K}_{i\bar{j}} = \begin{pmatrix} \mathcal{K}_{b\bar{b}} & \mathcal{K}_{b\bar{s}} \\ \mathcal{K}_{s\bar{b}} & \mathcal{K}_{s\bar{s}} \end{pmatrix} = \begin{pmatrix} \frac{3}{4\tau_b^2} & -\frac{9\tau_s^{\frac{1}{2}}}{8\tau_b^{5/2}} \\ -\frac{9\tau_s^{\frac{1}{2}}}{8\tau_b^{5/2}} & \frac{3}{8\tau_s^{\frac{1}{2}}\tau_b^{3/2}} \end{pmatrix}, \quad (\text{A.2})$$

and

$$\mathcal{K}_{i\bar{j}}^{-1} = \begin{pmatrix} \frac{4\tau_b^2}{3} & 4\tau_b\tau_s \\ 4\tau_b\tau_s & \frac{8\tau_b^{3/2}\tau_s^{1/2}}{3} \end{pmatrix}. \quad (\text{A.3})$$

In deriving this it is necessary to recall that  $\frac{\partial}{\partial\tau_b} = \frac{1}{2} \frac{\partial}{\partial T_b}$ . To compute the mass matrix we need to evaluate the second derivatives of the potential at the minimum. We start with

$$V = \frac{a_s^2 \lambda \sqrt{\tau_s} e^{-2a_s\tau_s}}{\tau_b^{3/2}} - \frac{\mu a_4 \tau_s e^{-a_4\tau_s} |W_0|}{\tau_b^3} + \frac{\nu |W_0|^2}{\tau_b^{9/2}}.$$

It can be shown after some computation that at the minimum of this potential,

$$\begin{aligned} e^{-a_s\tau_s} &= \left( \frac{\mu}{2\lambda} \right) \frac{|W_0|}{\tau_b^{3/2}} \frac{\sqrt{\tau_s}}{a_s} \left( 1 - \frac{3}{4a_s\tau_s} - \frac{3}{(4a_s\tau_s)^2} + \dots \right), \\ \tau_s^{3/2} \left( \frac{\mu^2}{4\lambda} \right) &= \nu \left( 1 + \frac{1}{2a_s\tau_s} + \frac{9}{(4a_s\tau_s)^2} + \dots \right). \end{aligned} \quad (\text{A.4})$$

Using the results of (A.4), we can show that to second order in an expansion in  $\epsilon = 1/(4a_s\tau_s)$ :

$$\frac{\partial^2 V}{\partial\tau_b^2} = \frac{9|W_0|^2\nu}{2\tau_b^{13/2}} \left( 1 + \frac{1}{2a_s\tau_s} \right), \quad (\text{A.5})$$

$$\frac{\partial^2 V}{\partial\tau_s^2} = \frac{2a_s^2|W_0|^2\nu}{\tau_b^{9/2}} \left( 1 - \frac{3}{4a_s\tau_s} + \frac{6}{(4a_s\tau_s)^2} \right), \quad (\text{A.6})$$

$$\frac{\partial^2 V}{\partial\tau_s\partial\tau_b} = -\frac{3a_s|W_0|^2\nu}{\tau_b^{11/2}} \left( 1 - \frac{5}{4a_s\tau_s} + \frac{4}{(4a_s\tau_s)^2} \right). \quad (\text{A.7})$$

The mass matrix is given by  $M_{i\bar{j}} = \frac{1}{2}V_{i\bar{j}}$ . This is

$$M_{i\bar{j}} = \begin{pmatrix} \frac{9|W_0|^2\nu}{4\tau_b^{13/2}} \left(1 + \frac{1}{2a_s\tau_s}\right) & -\frac{3a_s|W_0|^2\nu}{2\tau_b^{11/2}} \left(1 - \frac{5}{4a_s\tau_s} + \frac{4}{(4a_s\tau_s)^2}\right) \\ -\frac{3a_s|W_0|^2\nu}{2\tau_b^{11/2}} \left(1 - \frac{5}{4a_s\tau_s} + \frac{4}{(4a_s\tau_s)^2}\right) & \frac{a_s^2|W_0|^2\nu}{\tau_b^{9/2}} \left(1 - \frac{3}{4a_s\tau_s} + \frac{6}{(4a_s\tau_s)^2}\right) \end{pmatrix}, \quad (\text{A.8})$$

and so

$$\mathcal{K}^{-1}M^2 = \frac{2a_s\langle\tau_s\rangle|W_0|^2\nu}{3\langle\tau_b\rangle^{9/2}} \begin{pmatrix} -9(1-7\epsilon) & 6a_s\langle\tau_b\rangle(1-5\epsilon+16\epsilon^2) \\ -\frac{6\langle\tau_b\rangle^{1/2}}{\langle\tau_s\rangle^{1/2}}(1-5\epsilon+4\epsilon^2) & \frac{4a_s\langle\tau_b\rangle^{3/2}}{\langle\tau_s\rangle^{1/2}}(1-3\epsilon+6\epsilon^2) \end{pmatrix}. \quad (\text{A.9})$$

## B. Integrals

### B.1 NFW Halo

For the NFW halo, the integral  $\int dx\rho(x)$  we wish to perform is

$$\rho_0 r_s^3 \int_0^\infty dx \frac{1}{\sqrt{(x-\alpha)^2 + \beta^2}(r_s + \sqrt{(x-\alpha)^2 + \beta^2})^2}, \quad (\text{B.1})$$

where

$$\begin{aligned} \alpha &= R_0 \cos b \cos l, \\ \beta^2 &= R_0^2(1 - \cos^2 b \cos^2 l). \end{aligned}$$

We can rewrite this as

$$\int_{-\alpha}^\infty dy \frac{1}{\sqrt{y^2 + \beta^2}(r_s + \sqrt{\beta^2 + y^2})^2}.$$

We split this integral up:

$$= \int_{-\alpha}^0 dy \frac{1}{\sqrt{y^2 + \beta^2}(r_s + \sqrt{y^2 + \beta^2})^2} + \int_0^\infty dy \frac{1}{\sqrt{y^2 + \beta^2}(r_s + \sqrt{y^2 + \beta^2})^2}.$$

We now let  $y^2 + \beta^2 = z^2$ , so

$$y = \begin{cases} -\sqrt{z^2 - \beta^2} & y < 0 \\ \sqrt{z^2 - \beta^2} & y > 0 \end{cases}.$$

The integral then becomes

$$= \int_\beta^{\sqrt{\alpha^2 + \beta^2}} \frac{dz}{\sqrt{z^2 - \beta^2}(r_s + z)^2} + \int_\beta^\infty \frac{dz}{\sqrt{z^2 - \beta^2}(r_s + z)^2}.$$

Writing  $z' = z + r_s$ , we obtain

$$= \int_{\beta+r_s}^{r_s+\sqrt{\alpha^2+\beta^2}} \frac{dz'}{z'^2 \sqrt{z'^2 - 2r_s z' + (r_s^2 - \beta^2)}} + \int_{\beta+r_s}^\infty \frac{dz'}{z'^2 \sqrt{z'^2 - 2r_s z' + (r_s^2 - \beta^2)}}.$$

This is now a standard integral which can be found in integral tables. The indefinite integral ( $\alpha'$  and  $\beta'$  have no relation to  $\alpha$  and  $\beta$  above)

$$\int \frac{dz'}{z'^2 \sqrt{z'^2 + \alpha' z' + \beta'}} = -\frac{\sqrt{z'^2 + \alpha' z' + \beta'}}{\beta' z'} + \frac{\alpha'}{2\beta' \sqrt{\beta'}} \ln \left[ \frac{2\sqrt{\beta'(z'^2 + \alpha' z' + \beta')} + \alpha' z' + 2\beta'}{z'} \right].$$

After some manipulation, the integral (B.1) becomes  $\rho_0 r_s^3 X(b, l)$ , where

$$\begin{aligned} X(b, l) \equiv & \frac{1}{r_s^2 - R_1^2(b, l)} \left( -1 - \frac{R_0^2 - R_1^2(b, l)}{R_0 + r_s} \right) \\ & - \frac{r_s}{(r_s^2 - R_1^2(b, l))^{3/2}} \ln \left[ \frac{r_s R_0 + R_1^2(b, l) - \sqrt{(r_s^2 - R_1^2(b, l))(R_0^2 - R_1^2(b, l))}}{R_1(b, l)(r_s + R_0)} \right] \\ & + \frac{r_s}{(r_s^2 - R_1^2(b, l))^{3/2}} \ln \left[ \frac{R_1(b, l)}{r_s - \sqrt{r_s^2 - R_1^2(b, l)}} \right], \end{aligned} \quad (\text{B.2})$$

and  $R_1(b, l) = \sqrt{R_0^2(1 - \cos^2 b \cos^2 l)}$ .

## B.2 Diffuse Background Emission

Diffuse emission arises from moduli decays throughout the history of the universe. At a time  $t$  before the present, the scale factor of the universe satisfied  $\frac{a_t}{a_0} = \frac{1}{1+z}$ . The moduli number density was thus larger,  $n_t(\chi) = n_0(\chi)(1+z)^3$ , and the number of decay events between time  $t$  and time  $t + dt$  was

$$N = n_t(\chi) \frac{dt}{\tau_\chi} = n_0(\chi)(1+z)^3 \frac{dt}{\tau_\chi}.$$

The photons produced by these decay redshift and now have energies between  $E_\gamma$  and  $E_\gamma + dE_\gamma$ , with  $E_\gamma = \frac{m_\chi}{2(1+z)} \equiv \frac{E'}{1+z}$ , where we define  $E' = \frac{m_\chi}{2}$ . As the universe is expanding, there is a current number density of photons

$$N_\gamma(E_\gamma) dE_\gamma = 2n_0(\chi)(1+z)^3 \frac{dt}{\tau_\chi} \times \frac{1}{(1+z)^3}$$

of photons with energies between  $E_\gamma$  and  $E_\gamma + dE_\gamma$ . Now, as  $E_\gamma = \frac{E'}{1+z}$ , we have

$$dE_\gamma = -\frac{E'}{(1+z)^2} \frac{dz}{dt} dt.$$

The relation between redshift  $z$  and time  $t$  is determined by the matter and dark energy content of the universe. For matter density  $\Omega_m$  and dark energy  $\Omega_\Lambda$ , we have

$$\frac{dt}{dz} = -\frac{1}{H_0} \frac{1}{(1+z)^{5/2}} \left[ \Omega_m + \frac{1 - \Omega_m - \Omega_\Lambda}{1+z} + \frac{\Omega_\Lambda}{(1+z)^3} \right]^{-\frac{1}{2}} \quad (\text{B.3})$$

$$= -\frac{1}{H_0} \frac{1}{(1+z)^{5/2}} f(1+z), \quad (\text{B.4})$$

where  $f(x) = \left[ \Omega_m + \frac{(1-\Omega_m-\Omega_\Lambda)}{x} + \frac{\Omega_\Lambda}{x^3} \right]^{-\frac{1}{2}}$ . We can therefore write  $dz = -\frac{H_0 dt(1+z)^{5/2}}{f(1+z)}$ . Using  $\frac{E'}{E_\gamma} = 1+z$ , we thus have

$$dt = \frac{E_\gamma^{\frac{1}{2}} f\left(\frac{E'}{E_\gamma}\right)}{(E')^{3/2} H_0} dE_\gamma. \quad (\text{B.5})$$

This relates the range in departure times of the photons and the energy differences they have now. Then

$$N_\gamma(E_\gamma) dE_\gamma = \frac{2n_0(\chi)}{\tau_\chi} \frac{E_\gamma^{\frac{1}{2}} f(E'/E_\gamma) dE_\gamma}{H_0(E')^{3/2}} \text{ photons with energies between } E_\gamma \text{ and } E_\gamma + dE_\gamma.$$

The energy density per unit volume in decay photons is

$$\rho_\gamma(E_\gamma) dE_\gamma = \frac{2n_0(\chi)}{\tau_\chi} \left( \frac{2E_\gamma}{m_\chi} \right)^{3/2} f\left(\frac{E'}{E_\gamma}\right) \frac{dE_\gamma}{H_0}, \quad (\text{B.6})$$

with a photon number density

$$N_\gamma(E_\gamma) = \frac{\rho_\gamma(E_\gamma)}{E_\gamma}. \quad (\text{B.7})$$

This converts into a photon flux at a detector observing a solid angle  $d\Sigma$  of

$$\mathcal{N} = (d\Sigma) \times \Delta_D \times (cdt) \times \frac{N_\gamma}{4\pi}. \quad (\text{B.8})$$

If we observe a solid angle  $d\Sigma$ , the number of photons arriving in time  $dt$  from the diffuse cosmic background is then

$$\mathcal{N} = \frac{1}{4\pi} (d\Sigma) \times (\Delta_D) \times (dt) \times \frac{2n_0(\chi)}{\tau_\chi} E_\gamma^{\frac{1}{2}} \left( \frac{2}{m_\chi} \right)^{3/2} f\left(\frac{E'}{E_\gamma}\right) \frac{c}{H_0} dE_\gamma. \quad (\text{B.9})$$

This now gives us the number of arriving photons,  $\text{s}^{-1}\text{sr}^{-1}\text{cm}^{-2}(\text{keV})^{-1}$ . We can if we wish write this in terms of  $\Omega_\chi$ , using

$$n_0(\chi) = \left( \frac{\Omega_\chi}{\Omega_m} \right) \frac{\rho_0}{m_\chi},$$

to obtain

$$I_\gamma(E) = \frac{d\Sigma}{4\pi} \times \Delta_D \times dt \times dE_\gamma \times \left( \frac{\Omega_\chi}{\Omega_m} \right) \frac{2\rho_0}{\tau_\chi m_\chi} E_\gamma^{\frac{1}{2}} \left( \frac{2}{m_\chi} \right)^{3/2} f\left(\frac{E'}{E_\gamma}\right) \frac{c}{H_0}.$$

$\rho_0$  is the present day dark matter density.

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